





ENGINEERING THERMODYNAMICS

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PREFACE

FOR years there has been an important demand for a text-book on thermodynamics which would be brief and concise, but at the same time so clearly written as regards explanation that students of *average* ability in our large technical schools could read it without difficulty. A professor of thermodynamics wrote recently as follows: "I like the idea of making the text largely self-explanatory. Too many books require the reading of several lines between every two lines of the text." This book has been prepared to meet this demand and in writing it the authors have kept in mind these requirements. Further, it has been the idea of the authors to make this book particularly suitable for use in the larger technical schools where it is possible to give special courses on the subjects of steam turbines, pneumatic machinery, internal combustion engines, refrigeration and pumping machinery. Usually in the courses on these subjects the advanced and special theory of thermodynamics as it relates to each of them is taken up with completeness. At present there is too much duplication of subject matter in the large volumes on thermodynamics now available, which are made to include the descriptive matter and the applications of these special subjects.

The authors are particularly indebted to Professor Roy B. Fehr of The Pennsylvania State College for invaluable assistance and criticisms in the preparation of this work. Acknowledgments are due also to President Ira N. Hollis of Worcester Polytechnic Institute; Professor Lionel S. Marks of Harvard University and Massachusetts Institute of Technology; Professor H. C. Anderson of the University of Michigan; Dr. S. A. Moss of the General Electric Company; Dr. William Kent of Montclair, N. J.; Professor A. M. Greene of Rensselaer Polytechnic In-

stitute; Professor A. L. Westcott of University of Missouri; Professor A. A. Atkinson of Ohio University; and Professors J. A. Bursley and C. H. Fessenden of the University of Michigan, for assistance in various ways.

A book on applied thermodynamics published privately by the late Professor H. W. Spangler and with the preparation of which one of the authors was intimately associated has been consulted freely in the preparation of the last chapters. This book, as a whole, includes many of Professor Spangler's ideas as regards subject matter to be included in a book of this kind.

Acknowledgment of the services of Messrs. W. M. Sides and H. J. Hartranft of State College, Pa., is due for assistance in preparation and proof-reading.

J. A. MOYER
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SYMBOLS

- A = area in square feet.
 $B.t.u.$ = British thermal units (= 778 ft. lbs.).
 C_p = specific heat at constant pressure in B.t.u. per pound per degree.
 C_v = specific heat at constant volume in B.t.u. per pound per degree.
 C = a general constant in equations of perfect gases.
 D = degrees of superheat.
 E = external work in B.t.u. per pound; also sometimes used to express efficiency, usually as a decimal.
 E_a = available energy in B.t.u. per pound.
 F = force in pounds.
 H = heat per pound in B.t.u.*
 H_{sup} = total heat of superheated steam, B.t.u. per pound.
 I_H = total internal energy of steam (above 32° F.) in B.t.u. per pound.
 I_L = internal energy of evaporation of steam in B.t.u. per pound.
 J = reciprocal of mechanical equivalent of heat = $\frac{1}{778}$ (use becoming obsolete).
 K = specific heat in foot-pound units.
 L = latent heat of evaporation in B.t.u. per pound.
 M = mass (pounds).
 P = pressure in general or pressure in pounds per square foot.
 Q = quantity of heat in B.t.u.
 R = thermodynamic constant for gases; for air it is 53.3 (in foot-pound units per pound.)
 T = absolute temperature, in Fahr. degrees = $460 + t$.
 V = volume in cubic feet, also specific volume and velocity in feet per second.
 $W. E.$ = Wärme Einheit = kilogram calorie.
 W = work done in foot-pounds.
 a = area in square inches.
 c = constant of integration.
 d = distance in feet.
 e = subscript to represent base of natural logarithms.
 g = acceleration due to gravity = 32.2 feet per second per second.
 h = heat of the liquid per pound in B.t.u. (above 32° F.).
 k = a constant.
 \log = logarithm to base 10.
 \log_e = logarithm to natural base e (Napierian).
 n = general exponent for V (volume) in equations of perfect gases, also sometimes used for entropy of the liquid in B.t.u. per degree of *absolute* temperature.

* In steam tables it is total heat above 32° F.

p = pressure in pounds per square inch.

q = sometimes used for heat of the liquid in B.t.u. per pound (above 32° F.).

r = ratio of expansion (see page 33), also sometimes used for latent heat of evaporation in B.t.u. per pound.

t = temperature in ordinary Fahr. degrees.

v = specific volume, in cubic feet per pound (in some steam tables).

w = weight per cubic foot = density.

x = quality of steam expressed as a decimal.

Δ = differential symbol.

γ = ratio of specific heats $\frac{C_p}{C_v}$.

ϕ = total entropy $\frac{dQ}{T}$.

θ = entropy of the liquid in B.t.u. per pound per degree of *absolute* temperature.

Engineering Thermodynamics

CHAPTER I

INTRODUCTION TO THE THEORY OF HEAT ENGINES

THE beginning of our knowledge of the theory of heat on a scientific basis is probably Black's doctrine of latent heat.* The important relations as to the equivalence of heat and work were not established until about the middle of the nineteenth century, long after the heat engines invented by Newcomen and Watt had found general use. The greatest development in the theory of heat as regards its use in engines started with the publications of **Carnot** in 1824, when he showed that work is performed by a heat engine in the same proportion as the absolute temperature of the working fluid is changed from a higher to a lower degree. The next important development in heat theory was made by **Joule** who showed by his experiments in 1843 that for every **unit of heat** there was an exact equivalent in **mechanical work**. Although these experiments and the conclusions from them were epoch-making, there was little attempt at accurate calculations for designing steam and other engines until **Regnault** in 1847 published his classical data on the properties of steam. In the next few years important contributions to our knowledge of what the ideal engine should be, particularly as regards its efficiency in the way of heat conversion, were made by **Rankine**, **Clausius**, and **Lord Kelvin**. It was undoubtedly Rankine's philosophical treatment of the subject, as presented in various books and papers of which he was the author, that gave steam engineers scientific methods for the development of new and improved designs.

Thermodynamics is that branch of engineering science which deals with the interconversion of heat and work.† The object

* *Encyclopedia Britannica*, 11th ed., vol. 4, page 18.

† In general, thermodynamics may be defined as that branch of physical science which treats of the effects produced by heat.

of the study of thermodynamics is to consider in the light of the most recent investigations, questions relating to the influence on the efficiency of heat engines of increased pressure, of higher vacuums and expansions, of higher superheats (meaning also a greater range of expansion), of jacketing cylinders, of using the working medium in several cylinders one after the other; that is, compounding instead of expanding it only in a single cylinder, of putting receivers between the cylinders, and of reheating the working medium as it passes through them from one cylinder to the next. All these subjects require the most careful consideration by the engineer, even though theoretical considerations are not always directly applicable, and the final results which determine our engineering practice are largely determined by the theoretical analysis of carefully conducted experiments. The student should, therefore, keep in mind that although the actual conditions existing as regards the operation of our heat engines are relatively complex, the exact theory of their action is of great practical value, and he should, for this reason, give the study of the theory of heat engines his best attention. Because, also, of this complexity in the action of engines the study of the theory underlying their operation becomes all the more essential, as it must serve as a guide in deciding what conditions are most important for securing the highest efficiency.

Another important service which the study of thermodynamics renders is that of showing us what **maximum efficiency is attainable** for any engine operating under a given set of conditions. It often happens that the enthusiastic inventor presents data showing results which indicate an efficiency very much better than we can obtain with any of our present types of engines. In such cases it requires usually only a very little calculation, starting from the fundamental theory of thermodynamics, to show conclusively that the results claimed are absolutely impossible.

In many cases this study is also useful to the engineer in checking up his own work. For example, in calculating the results of a test made on a gas engine of the ordinary types, if it is found

that the efficiency as computed and reported is higher than 50 per cent, then obviously our knowledge of thermodynamics, and of the maximum possible efficiency of such engines operating under ordinary conditions, shows that the results are impossible and absurd. **The ability to interpret correctly the results of experiments performed on all kinds of heat engines and similar apparatus requires a thorough knowledge of the basic principles of thermodynamics.**

Laws of Thermodynamics. The heat engine in its simplest form does work by utilizing heat developed in a furnace or generated by the combustion of fuel within the engine itself. A part of the heat thus supplied is spent in doing mechanical work so that it no longer exists in the form of heat energy, while the remainder is rejected by the engine, still in the form of heat. The heat action thus described depends on two fundamental laws of thermodynamics called generally the First and Second.

First Law of Thermodynamics. The statement of the first of these is that the amount of heat which disappears in the action of the heat engine is proportional to the amount of mechanical work performed by the engine. In other words this law is nothing more than a simple statement of the **conservation of energy** as regards the equivalence of mechanical work and heat. With more exactness this first law may be stated as follows:

A definite quantity of heat goes out of existence for every unit of mechanical work that is performed, and, conversely, when heat is developed by the performance of mechanical work, a definite quantity of heat comes into existence for every unit of work.

Second Law of Thermodynamics. A self-acting machine cannot transmit heat from one body at a lower temperature to another body at a higher temperature unless aided by some external agency; that is, "heat cannot pass from a cold body to a hot body by a purely self-acting process" (Clausius). This law really states as regards heat engines the limits to their possible performance, which would be otherwise unlimited, if only

the "first law" is considered. It states further, as will be discovered in the discussion that follows, that no heat engine converts or can convert into work all of the heat supplied to it. A very large part of the heat supplied is necessarily rejected by the engine in the form of unused heat.

ENGINEERING UNITS

Engineers in English-speaking countries use as the standard of measurements the units generally known as the **foot-pound-second-system**. In this system the unit of length is the **foot**, the unit of mass is the **pound** (equivalent to 0.4535 kilogram), and the unit of time is the **second**. On the same basis we use as derived units the **square foot** as the unit of area in all theoretical calculations, unless it is expressly stated that the area is given in some other units, as for example in square inches. Practically all equations and formulas are stated with the volume given in **cubic feet**. Unit pressure, or what we call briefly "**pressure**," is the total applied force in pounds divided by the total area in square feet over which it is exerted, and is then expressed in terms of pounds per square foot.

Specific Volume is the term applied to the volume in cubic feet of a pound of a substance.

Density is the mass in pounds of a cubic foot of a substance. It is, therefore, the reciprocal of the specific volume.

For engineering purposes the ordinary unit of work is the **foot-pound**, which is the quantity of work performed by a force of one pound in moving through a distance of one foot. Quantities of heat are usually expressed in terms of the **British thermal unit (B.t.u.)**. This unit is the quantity of heat required to raise the temperature of **one pound** of water one degree on the Fahrenheit scale. The corresponding unit of heat on the Centigrade scale, used almost universally by physicists and chemists, is called the calorie (French and English), and *Wärme Einheit* (German).^{*} It is the quantity of heat required to raise the

^{*} Temperatures in Centigrade degrees are converted into Fahrenheit by multiplying by $\frac{9}{5}$ and adding 32. Kilogram-calories multiplied by 3.968 give the

temperature of **one gram** of water one degree on the Centigrade scale.

To make the definition of the B.t.u. accurate it is necessary to state at what temperature the rise of 1° F. as stated is to occur, because the specific heat of water is slightly variable. This is also one of the reasons for some of the variations in the tables of the properties of steam and other vapors that we shall observe in our calculations. Some tables are based on the assumption that the B.t.u. is the amount of heat required to raise the temperature of water 1° F. at the condition of maximum density of water, that is, between 39° and 40° F. Other tables are based on the amount of heat required to raise the temperature 1° F. from 60 to 61 degrees. Still another table, which is the latest and is generally considered to be the most accurate, uses for the B.t.u. one one-hundred-and-eightieth ($\frac{1}{180}$) of the amount of heat required to raise the temperature of water from 32° to 212° F. In other words, according to this last definition the B.t.u. is the average value of the amount of heat required to raise the temperature of one pound of water one degree between the conditions of freezing and boiling at atmospheric pressure.

Mechanical Equivalent of Heat. It has already been stated that for every heat unit there is a corresponding exact equivalent which can be expressed in the mechanical units of work. The number of work units corresponding to 1 B.t.u. is called the **mechanical equivalent of heat**. This quantity was necessarily determined by experiments. The most reliable results show that the average value of the mechanical equivalent of heat is 778 foot-pounds. This means, in other words, that a given number of B.t.u. multiplied by 778 gives the number of foot-pounds of work corresponding to the amount of heat as expressed by the thermal units.

This value of the mechanical equivalent of heat is the one given by Rowland, Griffith, Osborne Reynolds and Morby.

equivalent British thermal units (B.t.u.), and kilogram-calories per kilogram times 1.8 give B.t.u. per pound. A "small" calorie, or gram-calorie, is one-thousandth as large as a kilogram-calorie.

The first determination of this factor was made by Joule. In 1843 he obtained the value of 772 foot-pounds. In later experiments he found the value to lie between 774 and 775. A great many modern physicists believe that the most exact value is somewhere between 779 and 780 foot-pounds. The value stated above as given by Rowland and others, that is, **778 foot-pounds**, is, however, the value most generally accepted and is invariably used in engineering calculations.

THERMAL EFFICIENCY OF A HEAT ENGINE

$$\text{Thermal efficiency} = \frac{\text{Heat converted into work}}{\text{Heat supplied to the engine}}.$$

Since only a part of the heat supplied to an engine can be converted into work, the above ratio is a fraction always less than unity.

Since heat is not a form of matter, it must be transmitted, in a physical sense, by the use of some medium which is called the **working substance**. It is this working substance that takes in

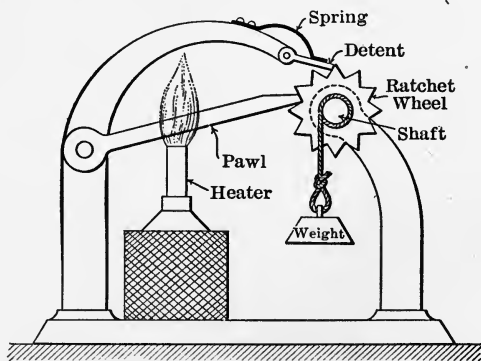


FIG. 1. — "Ratchet and Pawl" Type of Heat Engine.

and rejects heat. It may be in the form of a gas, a liquid or a solid. In general, there is a considerable change of volume in a suitable working substance for heat engines as that substance does work in overcoming resistance.

As the simplest imaginable example of a heat engine in which the working substance is neither a gas nor a liquid, but a solid material, we may think of the **working substance** of the engine as being a long rod of brass arranged to operate as the pawl of a ratchet wheel (**Fig. 1**) with teeth relatively close together. Now if the pawl is heated, it will elongate sufficiently to drive the tooth of the ratchet wheel with which it is in contact far enough forward to have the wheel held in its last position by a click or detent. When the pawl cools, as, for example, by pouring on cold water, it will contract and fall into the position where it will engage the next succeeding tooth on the wheel, which upon application of heat will again be driven forward and held in position, while the contracting of the pawl is again repeated. In fact, if the ratchet wheel referred to above is fastened at its center to a round shaft, its movement can be readily made to do work by raising the weight attached to a cord winding on the shaft. This heat engine has heat supplied to it at a high temperature. A small part of this heat is transformed into mechanical work in moving the ratchet wheel and with it the weight to be lifted, but by far the greatest part of the heat supplied is rejected in being absorbed by the water used for cooling and required to bring about the necessary contraction of the brass rod which is the working substance. This action as described here for the metallic working substance is typical of the action of all heat engines. They must take in heat at a relatively high temperature and reject it at a comparatively low temperature, and in the process they convert a small amount of heat supplied into mechanical work. This process is also somewhat similar to the performance of a water wheel which does work in bringing water from a higher to a lower level. In the case of the water wheel we say that the water has lost potential energy in dropping from a higher to a lower level. Similarly the working substance in heat engines loses heat energy, which is in fact (lost) potential energy as it drops from a higher to a lower temperature.

PROBLEMS

1. One pound of fuel has a heating value of 14,500 B.t.u. How many foot-pounds of work is it capable of producing, if all this heat could be converted into work?
Ans. 11,281,000 ft.-lbs.
2. An engine developed 15,560 ft.-lbs. of work. How much heat was required?
Ans. 20 B.t.u.
3. A stone weighing 2 lbs. is let fall from a height of 389 ft. into 1 lb. of water. How many degrees is the water raised?
Ans. 1° F.
4. A heat engine receives 100,000 B.t.u. of heat in the form of fuel, and during the same period 30,000 B.t.u. are converted into work. What is the thermal efficiency of the engine?
Ans. 30 per cent.
5. A gas engine receives 20,000 B.t.u. of heat in the form of fuel, and during the same period 3,112,000 ft.-lbs. of work are developed. What is the thermal efficiency of the engine?
Ans. 20 per cent.
6. Show that kilogram-calories per kilogram $\times 1.8$ give B.t.u. per pound.
7. Convert -40° C. into degrees Fahrenheit.
Ans. -40° F.

CHAPTER II

PROPERTIES OF PERFECT GASES

In the study of thermodynamics we shall have to deal mostly with fluids in the condition of a perfect gas, or a vapor. When the word "gas" is used it refers to what is more properly called a **perfect gas**, which is a fluid remaining in the gaseous condition even when subjected to moderately high pressures and low temperatures. Oxygen, hydrogen, nitrogen, air, and carbon dioxide are examples of what are called perfect gases. They are fluids which require a very great reduction in temperature and increase in pressure to bring them to the liquid state. **Vapors**, on the other hand, are fluids which are readily transformed into liquids by a very moderate reduction in tempera-

ture or increase in pressure. Common examples of vapors with which engineers have to deal are steam and ammonia.

For the present all our studies will be confined to the consideration of the properties of the perfect gases.

Relation between Pressure and Volume of a Perfect Gas. In practically all heat engines, work is done by changes of volume of a fluid, and the amount of work performed depends only on the relation of pressure to volume during such change and not at all on the form of the vessel containing this fluid. Since a good understanding of pressure and volume relations is most important, an illustration with a practical example will not be out of place.

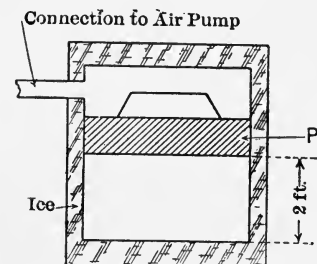


FIG. 2.—Constant Temperature Apparatus for Demonstrating Relation between Pressure and Volume of a Gas.

Fig. 2 shows a vessel filled with a perfect gas and surrounded

by a jacket filled with cracked ice. Its temperature will, therefore, be at 32° F. This vessel has a tightly fitting piston **P** of which the lower flat side has an **area of one square foot**. In the position shown the piston is two feet from the bottom of the vessel so that the volume between the piston and the bottom of the vessel is **two cubic feet**. The pressure on the gas is that due to the piston and the weights shown. Assume this total weight is 100 pounds and that the air pump connected to the top of the vessel maintains a practically perfect vacuum above the piston. Then we say the pressure on the gas below the piston is 100 pounds per square foot. If now the weights are increased to make the pressure on the gas 200 pounds per square foot the piston will sink down until it is only one foot from the bottom of the vessel, provided the ice keeps a constant temperature.*

Similarly, if the weight on the gas were 50 pounds and the vessel were made high enough, the lower side of the piston would be four feet from the bottom of the vessel. Examination of these figures shows that for all cases the **product of pressure and volume** is constant, and in this particular case is always 200 foot-pounds. Similarly, if the volume is expressed by V in cubic feet per pound and the pressure by P in pounds per square foot the product is a constant quantity, which for air at 32° F. is 26,220 foot-pounds. These facts are expressed by **Boyle's Law**, which states that the **volume** of a given mass of gas **varies inversely with the pressure**, provided the temperature is kept constant; that is, V varies inversely as P or

$$PV = \text{a constant}, \quad (1)$$

and also, if P_1 and V_1 represent some initial condition of pressure and volume and P_2 and V_2 are corresponding final conditions, then

$$P_1V_1 = P_2V_2 = \text{a constant}. \quad (1a)$$

* If the temperature is not maintained constant, because of the tendency of gases to expand with increase in temperature, it will be necessary to apply a total weight greater than 200 pounds to reduce the volume to one cubic foot.

Experiment shows that a permanent gas when heated increases in pressure if the volume is kept constant, and likewise increases in volume if the pressure is kept constant. If the pressure of a gas is kept constant and the temperature is raised 1° F. the gas will expand $\frac{1}{492}$ * of its volume at 32° F.; in other words, a volume of 492 cubic inches of gas will expand at constant pressure to 493 cubic inches when the temperature is raised from 32° to 33° F.

Similarly, this same **volume will be doubled** if the gas is heated 492 degrees (i.e., to 524° F.) at a constant pressure; and on the other hand, if the **volume remains constant** and the pressure is allowed to vary, the same increase in temperature (492 degrees) will double the pressure which the gas had initially.

Conversely, if the same law could be held to apply at very low temperatures, at 492° below 32° F. or -460° F. (-273° C.)† the volume would be zero.

Absolute Zero of Temperature. The examples above show that if temperatures are calculated, not from the ordinary zero but from 460 degrees below the ordinary Fahrenheit zero, the **volume** of the gas, if kept at **constant pressure**, will be proportional to the temperature reckoned from that zero point; and similarly, if the volume is kept constant, the **pressure** will be proportional to the temperature above that zero, which is called the **absolute zero**. Temperatures measured from the absolute zero as a basis are called **absolute temperatures**.

Absolute Temperature Conversion. If we represent the temperatures on the ordinary Fahrenheit scale by t and the corresponding **absolute** temperatures by T then

$$T = t + 460. \quad (2)$$

* This value is probably not exact. Various authorities give the following values: (1) $\frac{1}{492}$, (2) $\frac{1}{491.6}$, (3) $\frac{1}{491.4}$, (4) $\frac{1}{493}$, (5) $\frac{1}{492.7}$, (6) $\frac{1}{491}$, etc. The value given above is probably the best average value and is certainly close enough for all engineering calculations.

† See end of chapter for a brief description of the latest low temperature research.

If t' is the temperature on the Centigrade scale and T' the corresponding absolute temperature then

$$T' = t' + 273.$$

The laws of thermodynamics illustrated by the foregoing examples, dealing with volume and pressure changes corresponding to temperature variations, may be stated in two short paragraphs as follows:

(1) Under constant pressure the volume of a given mass of gas varies directly as the absolute temperature.

(2) Under constant volume the absolute pressure of a given mass of gas varies directly as the absolute temperature.

These fundamental principles, often called **Charles' Laws**, may also be stated thus:

$$\text{With pressure constant, } \frac{V_1}{V_2} = \frac{T_1}{T_2}, \quad (3)$$

$$\text{With volume constant, } \frac{P_1}{P_2} = \frac{T_1}{T_2}, \quad (4)$$

where V_1 and V_2 are respectively the initial and final volumes, P_1 and P_2 are the initial and final absolute pressures, and T_1 and T_2 are the absolute temperatures corresponding to the pressures and volumes of the same subscripts.

The following problem shows applications of Charles' laws:

A gas has a volume of 2 cubic feet, a pressure of 14.7 pounds per square inch absolute and a temperature of 60° F.

(a) What will be the volume of this gas if the temperature is increased to 120° F., the pressure remaining constant?

(b) What will be the pressure if the temperature is increased as in (a) but the volume remain constant?

Solution. (a) Since pressure remains constant and the substance is a gas, the volume varies directly as the absolute temperature.

Letting V_1 and T_1 be the initial conditions and V_2 and T_2 be the final conditions, then

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{2}{V_2} = \frac{60 + 460}{120 + 460},$$

$$V_2 = 2.33 \text{ cubic feet.}$$

(b) Since the volume remains constant,

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{14.7}{P_2} = \frac{60 + 460}{120 + 460}.$$

$$P_2 = 16.39 \text{ pounds per square inch absolute.}$$

Absolute Pressure.* The ordinary types of pressure gages used in engineering work do not measure the pressure from a true basis for comparison. Such instruments measure only the **excess** of pressure above that of the atmosphere. The pressure indicated by the instrument is called gage pressure, and may be expressed as a "true" or **absolute pressure** by adding the actual pressure of the atmosphere as obtained from the reading of a barometer. In all thermodynamic calculations in this book, unless it is expressly stated that gage pressures are meant, it is to be understood that absolute pressures are to be used. Boyle's and Charles' laws hold only for cases where the pressure P is in **absolute** terms. Furthermore, the same statement applies to all rational formulas in thermodynamics because all have their fundamental basis on these laws.

Gas Thermometers. If temperatures are measured by thermometers filled with a permanent gas like oxygen, hydrogen, air, or any other permanent gas used as the expansive medium, the increments of expansion are proportional to the temperature

* It is frequently necessary to reduce the pressures in inches of mercury or of water to the equivalent in pounds per square inch. Since the weight of a cubic inch of mercury at 70° F. is 0.4906 pound and of water at the same temperature is 0.0360 pound, pressures in inches of mercury at the usual "room" temperatures can be reduced to pounds per square inch by multiplying by 0.491 or by dividing by 2.035; and similarly, inches of water can be converted to pounds per square inch by multiplying by 0.0360 or by dividing by 27.78. Centimeters of mercury are reduced to pounds per square inch by multiplying by 0.1903.

Kilograms per square centimeter are reduced to pounds per square inch by multiplying the kilograms per square centimeter by 14.223 or by dividing by 0.0703. Grams divided by 28.35 are ounces avoirdupois, or one gram is approximately $\frac{1}{3\frac{1}{2}}$ ounce.

A cubic foot of water at 70° F. weighs 62.3 pounds and at 32° F. 62.4 pounds. At the ordinary room temperature the pressure due to 2.31 feet of water is equivalent to one pound per square inch.

measured from the absolute zero. The **air thermometer**, sometimes used by physicists, is one having the bulb and tube filled with air in such a way as to exert pressure upon a mercury column in a manometer or "U-tube" attached to the end of the air tube. Expansions and contractions in volume of the air in the bulb will change the relative levels of the mercury in the manometer, which can be graduated to indicate temperatures.

Combination of Boyle's and Charles' Laws. Equations (1), (3) and (4) cannot often be used in actual engineering problems as they stand, because it does not often happen that any one of the three variables (P , V and T) remains constant. A more general law must be developed, therefore, allowing for variations in all of the terms P , V and T . This is accomplished by combining equations (1a), (3) and (4). It will be assumed that we are dealing with a **pound** of gas of which the **initial** conditions of pressure, volume and temperature are represented by P_1 , V_1 and T_1 , while the corresponding **final** conditions, given by P_2 , V_2 and T_2 , are arrived at in **two steps**. The first step is in changing the volume from V_1 to V_2 and the pressure from P_1 to some intervening pressure P_2' while the temperature T_1 remains constant. This change can be expressed by Boyle's law (equation 1a).

With constant temperature (T_1),

$$\frac{V_1}{V_2} = \frac{P_2'}{P_1}, \quad (5)$$

from which, by solving,
$$P_2' = \frac{P_1 V_1}{V_2}, \quad (6)$$

where P_2' is the resulting pressure of the gas when its volume is changed from V_1 to V_2 , with the temperature remaining constant at T_1 .

The second step is in the change in pressure from P_2' to P_2 and the temperature from T_1 to T_2 , while the volume remains constant at V_2 . This step is expressed by equation (4) as follows:

With constant volume (V_2),

$$\frac{P_2'}{P_2} = \frac{T_1}{T_2}, \quad (7)$$

which may be written
$$P_2 = \frac{P_2' T_2}{T_1}. \quad (8)$$

Substituting now the value of P_2' from (6) in (8), we have

$$P_2 = \frac{P_1 V_1 T_2}{V_2 T_1}, \quad (9)$$

which may be arranged to read,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}. \quad (10)$$

These steps are shown diagrammatically in **Figs. 3** and **4**. In **Fig. 3** a surface is shown in which the lines indicating these changes lie.

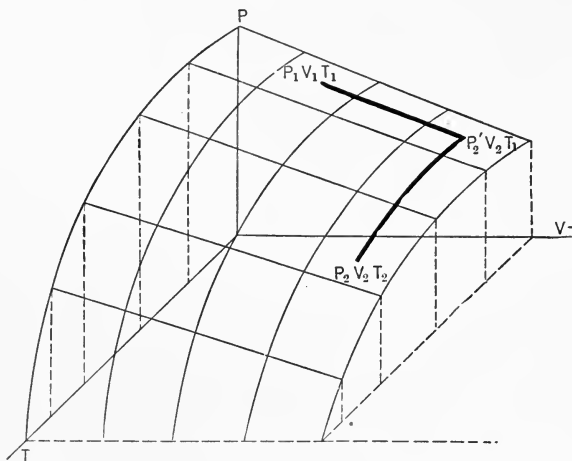


FIG. 3.—“Surface” Diagram Illustrating Derivation of “Combination” Law of Gases.

The following problem shows the application of equation 10:

A quantity of air at atmospheric pressure has a volume of 2000 cubic feet when the barometer reads 28.80 inches of mercury and the temperature is 40° C. What will be the volume

of this air at a temperature of 0° C. when the barometer reads 29.96 inches of mercury?

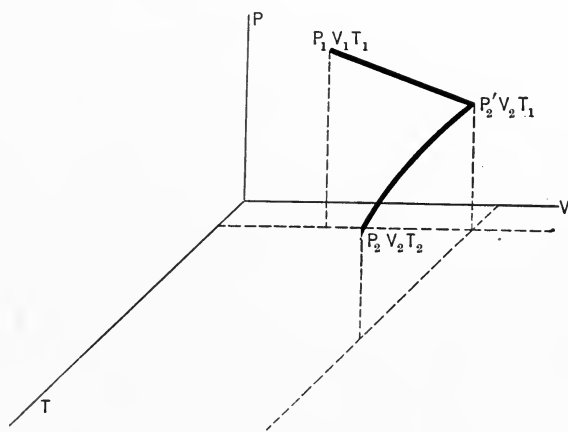


FIG. 4.—Isometric Drawing Illustrating Derivation of “Combination” Law of Gases.

Solution. Volume, pressure and temperature vary in this case as in the following equation,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}.$$

Letting

P_1, V_1, T_1 = initial conditions,

P_2, V_2, T_2 = final conditions,

then

$$\frac{28.80 \times 2000}{40 + 273} = \frac{29.96 \times V_2}{0 + 273},$$

$$V_2 = 1676.42 \text{ cubic feet.}$$

Now, since P_2, V_2 and T_2 in equation (10) are any other **simultaneous conditions** of the gas, we may also write the following more general relations:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} = \text{a constant}, \quad (10')$$

and, therefore,

$$PV = RT, \quad (11)$$

where **R** is the “**gas constant.**”

This constant is a most important quantity in thermodynamic calculations, its value varying only with the kind of gas dealt with. It must be remembered that equation (11) was derived for one pound of gas, and therefore, in order to make it applicable for any mass of gas, it is only necessary to multiply the constant R by the mass M , the volume V in the equation being the volume corresponding to the mass M . We thus obtain the final form of the equation representing the “combination law” of perfect gases:

$$PV = MRT, \quad (11')$$

where P = absolute pressure in pounds per square foot,

V = actual volume in cubic feet,

M = mass of gas in pounds,

R = the “gas constant” for one pound of gas in foot-pound units,

T = the absolute temperature in Fahrenheit degrees.

This equation is applicable to any perfect gas within the limits of pressure and temperature employed in common engineering practice. The “thermodynamic” state of a gas is known when its pressure, volume, temperature, mass and composition* are known, and when any four of these quantities are known the fifth can be found by equation (11').

Values of the constant R in the table on page 23 have been calculated from experimental data of the specific volumes of the various gases mentioned. Thus, the specific volume of air is given as 12.38 cubic feet per pound at 32° F. (492 degrees absolute) and at atmospheric pressure ($14.7 \times 144 = 2117$ pounds per square foot). Substituting these data in equation (11'), we have

$$\begin{aligned} 2117 \times 12.38 &= 1 \times R \times 492, \\ R &= 53.3 \text{ (nearly).}^\dagger \end{aligned}$$

It is also useful to observe that the value of R for **any** gas

* The gas constant R depends upon the composition of the gas.

† The tables on page 23 were calculated from slightly more exact data, but the agreement shown is good enough for all engineering calculations.

can be approximately calculated by dividing the number 1544 by its molecular weight. For example, the value of R for acetylene gas (C_2H_2), having a molecular weight of $24 + 2$ or 26, is $1544 \div 26$ or 59.4, which is the value commonly given for this gas.

This latter method gives a very rapid and sufficiently exact means for determining the value of R of any gas for which the chemical formula is known.

The following problem shows the application of equation 11':

What is the volume of a tank that will hold a mass of 5 pounds of air when the pressure is 200 pounds per square inch absolute and the temperature $40^\circ C$?

Solution. Three unknowns are given and the fourth is to be found by the equation,

$$P_1 V_1 = M_1 R_1 T_1.$$

Changing values given to the proper units,

$$\begin{aligned} 200 \text{ pounds per square inch absolute} &\times 144 \\ &= 28,800 \text{ pounds per square foot absolute.} \\ 40^\circ C. \times \frac{9}{5} + 32 &= 104^\circ F. \end{aligned}$$

Then substituting in equation 11',

$$\begin{aligned} 28,800 \times V_1 &= 5 \times 53.3 (104 + 460), \\ V_1 &= 5.22 \text{ cubic feet.} \end{aligned}$$

Heat and Its Effect on Expansion. It is most important to discuss here, before going on to the more difficult problems having to do with the expansions of gases, the most essential considerations involved in this study. In general the effect of adding heat to a gas is to raise its temperature. If heat is added so that at the same time the gas expands and does work its temperature may either rise or fall according as the amount of heat added is greater or less than the heat equivalent of the external work done. The following concise statement always applies, and will be found very useful:

Heat added = increase in internal energy + external work (12)

Specific Heat. The amount of heat required to raise the temperature of a unit mass of a substance one degree is called its

specific heat. In the English system the specific heat is the number of British thermal units (B.t.u.) required to raise the temperature of a pound of the substance 1° F.

The specific heat of gases and vapors changes considerably in value according to the conditions under which the heat is applied. If heat is applied to a gas or a vapor held in a closed vessel, with no chance for expansion, no work is performed, and, therefore, all the heat added is used to increase the temperature. This is the condition in a boiler, for example, when no steam is being drawn off. In this case we use the symbol C_v to represent the **specific heat** during its application at **constant volume**. If, on the other hand, the heating is done while the pressure is kept constant and the volume is allowed to change to permit expansion and the performance of work,* we use the symbol C_p to represent the **specific heat** during its application at **constant pressure**. Heat application at constant pressure is the condition that is most interesting to the engineer. When his engines are running his boilers are making steam at **constant pressure**. The heat energy absorbed by a pound of a substance for raising **only the temperature** must certainly be the same regardless of the conditions of pressure and volume. Since, however, for constant pressure conditions some external work is always done, requiring a correspondingly larger amount of heat energy than for the case when the volume is constant, it follows that C_p is always greater than C_v . In other words, C_p is equal to C_v plus the heat equivalent of the work done by one pound of the substance in expanding at constant pressure, while the temperature is raised 1° F., if C_p and C_v are in B.t.u.

Specific heat can also be expressed in units of work (foot-pounds). When C_p and C_v are expressed in B.t.u. then the corresponding values in foot-pounds will be $778 C_p$ and $778 C_v$.

External Work. The external work or the work done by a gas in its expansion is represented graphically by **Fig. 5**. This

* Work performed as the result of expansion of a gas or vapor is always done at the expense of an equivalent amount of heat energy.

is the simplest sort of figure to show that the area under the expansion line **BC** is proportional to the work done in the expansion.

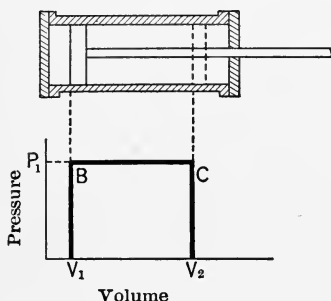


FIG. 5. — External Work by Expansion.

Let us represent the initial condition of the gas at **B** as regards pressure and volume by P_1 and V_1 and the final condition at **C** by P_1 and V_2 (expansion being at constant pressure), then elementary mechanics teaches that the force P moved through a distance represented by the difference of abscissas ($V_2 - V_1$) is a measure of the work done. Obviously the area under the line **BC** divided by the

horizontal length ($V_2 - V_1$) is the average value of the force P . If, further, and in general, we represent the area under **BC** by the symbol A , then we can write,

$$\frac{A}{V_2 - V_1} = \text{average value of } P,$$

whether or not P is constant. And also,

$$\text{Work done} = \frac{A}{V_2 - V_1} \times (V_2 - V_1) = A.$$

The same principle applies whether the line **BC** is a straight line as shown or a very irregular curve, as will be shown later.

Internal Energy. The heat energy possessed by a gas or vapor, or, we may say, the heat energy which is in a gas or vapor in a form similar to “potential” energy, is called its **internal energy**.* Thus, an amount of heat added to a substance when no work is performed is **all** added to the internal energy of that substance. On the other hand, when heat is added while work is being performed, the **internal energy is increased** only by the difference between the heat added and the work done.

Internal energy may also be defined as the energy which a

* It is also sometimes called *intrinsic energy*, since it may be said to “reside” within the substance and has not been transferred to any other substance.

gas or vapor possesses by virtue of its **temperature**, and may be expressed as follows:

$$\text{Internal energy} = C_v T \text{ (in B.t.u.)},$$

where T is the absolute temperature and C_v the specific heat at constant volume. From the paragraph on specific heat it will be remembered that C_v takes into account only that heat required to raise the temperature, since under constant volume conditions no external work is done; and therefore, in dealing with internal energy, since we do not care anything about the external work that may have been done, C_v is always used.

It is with the **change** in internal energy that we are generally concerned, and this change is, obviously,

$$\begin{aligned} &\text{Increase in internal energy} \\ &= C_v (T_2 - T_1) \text{ in B.t.u. (for one pound of gas). } \quad (13) \end{aligned}$$

This formula will always apply, for in whatever way the temperature of a substance is changed from T_1 to T_2 the change in its internal energy is the same since, by definition, it depends only upon the temperature.

Joule's Law. In the case of ideally perfect gases such as these thermodynamic equations must deal with, it is assumed, when a gas expands* without doing external work and without taking in or giving out heat (and, therefore, without changing its stock of internal energy) that its temperature does not change.

Relation of Specific Heats and Gas Constant. In order to derive this relation we must consider certain fundamental equations in thermodynamics and in general physics.

A formula in physics for the total heat added, H , is as follows:

$$H = \text{mass} \times \text{specific heat} \times \text{difference in temperature.}$$

* It was for a long time supposed that when a gas expanded without doing work, and without taking in or giving out heat, that its temperature did not change. This fact was based on the famous experiments of Joule. Later investigations by Lord Kelvin and Linde have shown that this statement is not exactly correct as all known gases show a change in temperature under these conditions. This change in temperature is known as the "Joule-Thomson" effect.

If the heat is added at constant pressure, then obviously, in B.t.u.,

$$H = MC_p (T_2 - T_1). \quad (14)$$

Also, by equation (13), the increase in internal energy when heat is added

$$= MC_v (T_2 - T_1). \quad (15)$$

Another formula in physics is stated as follows:

Work (external) = force \times distance = Fd

= PAd , where F = force in pounds,

d = distance moved in feet,

P = unit pressure in pounds per square foot,

and

A = area in square feet.

Combining the factors A and d we obtain the change of volume,

$Ad = \Delta v$, where Δv is the change in volume.

Therefore,

External work = $P\Delta v$.

When the pressure remains constant, equation (15) may be expressed as follows:

$$\begin{aligned} \text{External work} &= P (V_2 - V_1), \text{ foot-pounds} \\ &= \frac{P (V_2 - V_1)}{778}, \text{ B.t.u.} \end{aligned} \quad (16)$$

Remember the fundamental equation (12), which is as follows:

Heat added = increase in internal energy + external work.

Substituting in the above equation (14), (15), and (16), we have

$$MC_p (T_2 - T_1) = MC_v (T_2 - T_1) + \frac{P (V_2 - V_1)}{778}. \quad (17)$$

By equation (11'),

$$PV_2 = MRT_2 \text{ and } PV_1 = MRT_1.$$

Substituting these values in (17),

$$MC_p (T_2 - T_1) = MC_v (T_2 - T_1) + \frac{MR (T_2 - T_1)}{778}. \quad (18)$$

Simplifying,

$$C_p = C_v + \frac{R}{778}. \quad (19)$$

Ratio of Specific Heats (γ). The ratio of the specific heat at constant pressure to the specific heat at constant volume enters into many thermodynamic equations and is of so much importance in engineering calculations that it is commonly represented by the symbol γ ; thus

$$\frac{C_p}{C_v} = \gamma. \quad (20)$$

From the previous discussion of the relative values of C_p and C_v it is evident that γ must always have a value greater than unity.

By means of equations (19) and (20), the relation between C_v , γ and R may be obtained:

$$C_p = C_v + \frac{R}{778}. \quad (a)$$

$$\frac{C_p}{C_v} = \gamma. \quad (b)$$

From (b), $C_p = \gamma C_v$.

Substituting in (a) $\gamma C_v = C_v + \frac{R}{778},$

$$C_v(\gamma - 1) = \frac{R}{778}.$$

Therefore, $C_v = \frac{R}{778(\gamma - 1)}. \quad (21)$

DATA FOR VARIOUS GASES

Name of gas.	Molecular weight.	Specific * volume, cu. ft. per lb.	Density,* lbs. per cu. ft.	Value of R per lb. (ft.-lb. units).	C_p	C_v	γ
Air (pure).....	29	12.38	0.0807	53.3	0.237	0.169	1.40
Carbon dioxide (CO ₂).....	44	8.17	0.1224	35.1	0.200	0.154	1.30
Carbon monoxide (CO)....	28	12.80	0.0779	55.2	0.243	0.173	1.40
Hydrogen (H ₂).....	2	179.65	0.0056	766.1	3.410	2.435	1.40
Marsh gas (CH ₄).....	16	22.45	0.0445	96.3	0.593	0.470	1.26
Nitrogen (N ₂).....	28	12.81	0.0779	55.1	0.244	0.173	1.41
Oxygen (O ₂).....	32	11.22	0.0891	48.3	0.217	0.155	1.40
Sulphur dioxide (SO ₂).....	64	5.61	0.1781	24.1	0.154	0.123	1.25

* Under standard conditions, i.e., 32° F. and 14.7 lbs. per sq. in. absolute pressure.

LOW TEMPERATURE RESEARCHES

In *Harper's Monthly Magazine** is described the remarkable achievement of Professor Kamerlingh Onnes, of Leyden, in approaching the absolute zero of temperature, viz., -273°C. , within 1.8 degrees, when using helium. This is 93 degrees colder than the temperature of liquid air. Professor Onnes in 1913 won the Nobel Prize for researches in the science of physics, and in 1914 cooled a coil of lead wire to a temperature where the electrical resistance becomes zero, the current, once started, continuing to flow without supply so long as its temperature is kept at about 270 degrees below 0°C.

At this low temperature many of the properties of matter at ordinary temperatures were expected to disappear, or at least to become greatly modified. The temperature of liquid air is -180°C. , and by its evaporation it is possible to reach -205°C. At these temperatures ordinary soft lead becomes hard and brittle like cast iron, rubber becomes fragile like glass, and alcohol can be frozen to a white solid and a candle can be made from it. The next lowest depression in temperature was made by Sir James Dewar, who, in liquefying hydrogen, attained a temperature of 253 degrees below 0°C. , only 20 degrees from absolute zero of temperature, but no startling effects were observed there.

A reduction in temperature by evaporating a liquid with a very low boiling point results from the consumption of the heat within the liquid, making it necessary to prevent the entrance of heat from the outside. This is accomplished, firstly, by evaporating the liquid in a vacuum-jacketed glass vessel having the heat-insulating property of a "thermos" bottle; secondly, by surrounding the vessel with a liquid of low temperature. Until recently the lowest temperature produced, about -260°C. , was obtained by evaporating liquid hydrogen under reduced pressure in a vessel surrounded by liquid air. This temperature was thought to be the lowest limit attainable, liquid hydro-

* Vol. 129 (1914), pages 783-9.

gen having a lower boiling point than any other substance then known.

However, when the peculiar and extremely rare gas known as helium was discovered, which is formed by the spontaneous decomposition of the element radium, the attainment of the absolute zero seemed feasible, provided a sufficient supply of helium could be obtained. This gas proved to be even more difficult to liquefy than hydrogen, its boiling point being lower. However, it requires about 1000 cubic feet of gas to form one cubic foot of liquid. Professor Onnes succeeded in building an apparatus which enables him now to produce about half a pint of liquid helium in a few hours. The glass vessel in which the helium is liquefied must be surrounded by a vessel containing liquid hydrogen, boiling under reduced pressure, which in turn must be surrounded by liquid air. The problem is comparable in difficulty to the construction of an apparatus for making ice inside of a furnace. Helium sells for about \$50 a quart, but by the generosity of an American private commercial source, Professor Onnes obtained an enormous quantity after searching European markets in vain.

Liquefaction of the gas helium is accomplished by means of an apparatus similar to that employed for the liquefaction of air, except that the compression pump, pipes and receiver must be absolutely gas-tight. Professor Onnes employs a pump with mercury pistons; every joint and valve in the system is immersed in a bath of oil, so that leaks will manifest themselves by small bubbles rising through the liquid. The apparatus is sufficiently perfect to permit the compression of helium to 3000 pounds per square inch. The compressed fluid is passed through pipes immersed in liquid air or hydrogen, then through the coiled tube of the liquefier, whereby a portion becomes liquid and drops into a glass vessel, while the remainder passes off through another pipe and returns to the compressor—the process repeating itself over and over again without any loss of gas.

At atmospheric pressure liquid helium boils at -268.6°C . or 4.4 degrees absolute; but by reducing the pressure in the tube

by means of an air pump, Professor Onnes has succeeded in reaching an actual measured temperature of only 1.8 degrees absolute. Such temperatures can be measured only by means of a gas thermometer filled with helium under very low pressure, instead of the usual hydrogen-filled thermometer.

PROBLEMS

1. The pressure in a gas tank is 90 lbs. per sq. in. by the gage. If atmospheric pressure is 15 lbs. per sq. in. absolute, what is the absolute pressure in the tank?

Ans. 105 lbs. per sq. in.

2. What is the absolute pressure within the above gas tank in pounds per square foot?

Ans. 15,120 lbs. per sq. ft.

3. The gas in Problem 1 is at 60° F. temperature. What is the absolute Fahrenheit temperature of this gas?

Ans. 520 degrees.

4. Air is at a temperature of 40° C. What is the absolute Fahrenheit temperature?

Ans. 564° absolute F.

5. Air at constant pressure has an initial volume of 2 cu. ft. and temperature of 60° F.; it is heated until the volume is doubled. What is the resulting temperature in degrees Fahrenheit?

Ans. $\left\{ \begin{array}{l} 1040^{\circ} \text{ absolute F.} \\ 580^{\circ} \text{ ordinary F.} \end{array} \right.$

6. Air is cooled under constant volume. The initial condition is pressure of 30 lbs. per sq. in. absolute and temperature of 101° F. The final condition has a temperature of 50° F. What is the final pressure?

Ans. 27.27 lbs. per sq. in. absolute.

7. One pound of hydrogen is cooled under constant pressure from a volume of 1 cu. ft. and temperature of 300° F. to a temperature of 60° F. What is the resulting volume?

Ans. 0.684 cu. ft.

8. A tank whose volume is 50 cu. ft. contains air at 105 lbs. per sq. in. absolute pressure and temperature of 80° F. How many pounds of air does the tank contain?

Ans. 26.26 lbs.

9. An automobile tire has a mean diameter of 34 in. and 4 in. width. It is pumped to 80 lbs. per sq. in. gage pressure at a temperature of 60° F.; atmospheric pressure 15 lbs. per sq. in. absolute.

(a) How many pounds of air does the tire contain?

Ans. 0.38 lb.

(b) Assuming no change of volume, what would be the gage pressure of the tire if placed in the sun at 100° F.?

Ans. 87 lbs. per sq. in.

10. An acetylene gas tank is to be made to hold 0.25 lb. of this substance when the pressure is 250 lbs. per sq. in. gage, atmospheric pressure 15 lbs. per sq. in. absolute, and the temperature of the gas 70° F. What will be the volume in cu. ft.?

Ans. 0.206 cu. ft.

11. A quantity of air at a temperature of 70° F. and a pressure of 15 lbs. per sq. in. absolute has a volume of 5 cu. ft. What is the volume of the same air when the pressure is changed at constant temperature to 60 lbs. per sq. in. absolute?

Ans. 1.25 cu. ft.

12. How many pounds of air are present in Problem 11?

Ans. 0.383 lb.

13. The volume of a quantity of air is 10 cu. ft. at a temperature of 60° F. when the pressure is 15 lbs. per sq. in. absolute. What is the pressure of this air when the volume becomes 60 cu. ft. and the temperature 60° F.?

Ans. 2.5 lbs. per sq. in. absolute.

14. How many pounds of air are present in Problem 13?

Ans. 0.779 lb.

15. A tank contains 200 cu. ft. of air at a temperature of 60° F. and under a pressure of 200 lbs. per sq. in. absolute.

(a) What weight of air is present?

Ans. 207.5 lbs.

(b) How many cubic feet will this air occupy at 15 lbs. per sq. in. absolute and temperature of 100° F.?

Ans. 2880 cu. ft.

16. The volume of a quantity of air at 70° F. under a pressure of 15 lbs. per sq. in. absolute is 20 cu. ft. What is the temperature of this air when the volume becomes 5 cu. ft. and the pressure 80 lbs. per sq. in. absolute?

Ans. 707° F. absolute.

17. Air at constant pressure has a specific heat of 0.237 B.t.u. How many B.t.u. are required to raise 2 lbs. from 60° to 100° F.?

Ans. 18.96 B.t.u.

18. Three pounds of a substance has 75 B.t.u. supplied to it to change the temperature 100° F. What is its specific heat?

Ans. 0.25 B.t.u.

19. If the specific heat of air under constant pressure is 0.237 and the value of R is 53.3, find the value of the specific heat under constant volume.

Ans. 0.169 B.t.u.

20. From the data in Problem 19 find the value of γ for air.

Ans. 1.41.

21. An auto tire has a volume of 0.66 cu. ft. and is to be pumped to a pressure of 85 lbs. per sq. in. gage at a temperature of 50° F. What volume tank will be required to inflate four such tires if the air can be stored in this tank at 250 lbs. per sq. in. gage pressure and temperature of 70° F.?

Suggestion. Assume the tires are filled with air at atmospheric pressure before starting to pump, and also when all tires have been inflated that the tank will contain its volume of air at the final pressure in the tires.

Let P_a = atmospheric pressure; P_t = gage pressure in tires; P_k = gage pressure in tank; * T = absolute temperature in tires or tank; V_t = volume of tires; V_k = volume of tank; M_a = mass (pounds) of air in tires when at atmospheric pressure; M_b = total mass (pounds) of air in tires when at the tire pressure; M_c = total mass (pounds) of air in tank at final tire pressure; M_d = total mass (pounds) of air in tank at the final tank pressure before beginning to fill the tires.

Then

$$\begin{aligned}(M_b - M_a) &= \text{Mass supplied tires} = M_d - M_c \\ (P_t + P_a) V_t &= M_b \times R \times T_t \\ \frac{P_a \times V_t}{P_t \times V_t} &= \frac{M_a \times R \times T_t}{(M_b - M_a) R T_t}\end{aligned}$$

Solving this for $(M_b - M_a)$:

$$\begin{aligned}(P_k + P_a) V_k &= M_d \times R \times T_k \\ \frac{(P_t + P_a) V_k}{(P_k - P_t) V_k} &= \frac{M_c \times R \times T_k}{(M_d - M_c) R T_k}\end{aligned}$$

Substitute the value of $(M_b - M_a)$ found above for $(M_d - M_c)$ and then solve for V_k .

Ans. $V_k = 1.41$ cu. ft.

22. How many B.t.u. are required to double the volume of 1 lb. of air at constant pressure from 50° F.; specific heat is 0.237 B.t.u.?

Ans. 121 B.t.u.

23. A tank filled with 200 cu. ft. of air at 15 lbs. per sq. in. absolute and 60° F. is heated to 150° F.

(a) What will be the resulting air pressure in the tank?

Ans. 17.6 lbs. per sq. in. absolute.

(b) How many B.t.u. will be required to heat the air?

Ans. 240 B.t.u.

24. A tank contains 200 cu. ft. of air at 60° F. and 40 lbs. per sq. in. absolute. If 500 B.t.u. of heat are added to it, what will be the resulting pressure and temperature?

Ans. $\begin{cases} 45.5 \text{ lbs. per sq. in. absolute.} \\ 131.3^\circ \text{ F.} \end{cases}$

* All pressures are in pounds per square foot.

CHAPTER III

EXPANSION AND COMPRESSION OF GASES

THE general equation in the form $PV = MRT^*$ for the expansion or compression of gases has three related variables, (1) **pressure**, (2) **volume** and (3) **temperature**. For a given mass of gas with any two of these variables given, obviously, the third is fixed. As regards the analysis of the action of heat engines, the pressure and volume relations are most important, and graphical diagrams, called pressure-volume or P - V diagrams, are frequently needed to assist in the analysis. The

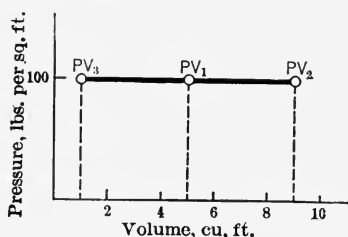


FIG. 6. — Diagram of Expansion and Compression at Constant Pressure.

simplest sort of diagram of this kind is shown in Fig. 6, in which the vertical scale of coördinates represents **pressures** and the horizontal, **volumes**.

Assume that the pressure and volume of a pound of a given gas are given by the coördinates P and V_1 , which are plotted in the middle of the diagram. It will be assumed further that the **pressure remains constant** in the changes to be indicated. Now if the gas is **expanded** until its volume becomes V_2 , then its condition as regards pressure and volume would be represented at PV_2 . If, on the other hand, the gas had been **compressed** while a constant pressure was maintained, its final condition would

* Equation (11'), p. 17.

be represented by the point PV_3 to the left of PV_1 . Similarly, any line whether straight or curved extending from the initial condition of the gas at PV_1 will represent an expansion when drawn in the direction away from the zero of volumes and will represent a compression when tending toward the same zero.

It is readily shown that areas on such diagrams represent the product of pressure and volume, and, therefore, work or energy (see page 20). Thus in **Fig. 6** the area under the curve PV_3 to PV_2 represents on the scales given 100 (pounds per square foot) \times (9 - 1) cubic feet or 800 foot-pounds irrespective of whether it is an expansion or a compression from the initial condition. When a series of curves are joined together to form a closed figure which shows the varying conditions of pressure and volume of a gas, the figure is called an **indicator diagram**.

Most of the lines to be studied in heat engine diagrams are either straight or else they can be exactly or approximately represented by an equation in the form

$$PV^n = \text{a constant}, \quad (22)$$

where the index n , as experimentally determined, has varying numerical values, but is almost invariably constant for any one curve. When the lines of the diagram are straight the areas of simple rectangles and triangles need only be calculated to find the work done, and for these cases no discussion is here necessary. The two most common forms of curves to be dealt with in expansions are (1) when there is expansion with addition of heat at such a rate as to maintain the temperature of the gas constant throughout the expansion. Such an expansion is called **isothermal**. The other important kind of expansion (2) occurs when work is done by the gas without the addition or abstraction of heat. To do this work some of the internal heat energy contained in the gas must be transformed in proportion to the amount of work done. Such an expansion is called **adiabatic**.

The following problems show the application of the foregoing theory to straight line expansion:

Case 1. One pound of air having an initial temperature of 60° F. is expanded to 100° F. under constant pressure. Find

- (a) External work during expansion;
- (b) Heat required to produce the expansion.

Solution. From the fundamental theory, the heat added equals the increase in internal energy plus the external work done. In solving then for the heat added or required during any expansion it is only necessary to find the external work (which is equal to the area under the expansion curve) and add to it the heat needed to increase the internal energy.

The external work $W = P_1 (V_2 - V_1)$ or its equivalent $MR(T_2 - T_1) = 1 \times 53.3 (100 + 460) - (60 + 460) = 2132$ ft.-lbs.

The increase in internal energy

$$\begin{aligned} &= MC_v (T_2 - T_1) \\ &= 1 \times 0.169 (100 + 460) - (60 + 460) \\ &= 6.75 \text{ B.t.u.} \end{aligned}$$

$$\text{Heat required} = 6.75 + \frac{2132}{778} = 9.50 \text{ B.t.u.}$$

As part of the data C_p is known. Then for this case of constant pressure expansion, the heat required equals by another method,

$$\begin{aligned} MC_p (T_2 - T_1) &= 1 \times 0.237 (100 + 460) - (60 + 460) \\ &= 9.5 \text{ B.t.u. approximately.} \end{aligned}$$

Case 2. One pound of air having an initial temperature of 60° F. is heated at constant volume until the final temperature is 120° F. Find

- (a) External work;
- (b) Heat required.

Solution. Applying the same theory as above,

Heat added = increase in internal energy + external work.

External work = 0, since there is no area under the curve.

Then

$$\begin{aligned} \text{Heat added} &= \text{increase in internal energy} + 0 \\ &= MC_v (T_2 - T_1) + 0 \\ &= 1 \times 0.169 (120 + 460) - (60 + 460) + 0 \\ &= 6.76 \text{ B.t.u.} \end{aligned}$$

Isothermal Expansion and Compression. In an isothermal expansion or compression the temperature of the working substance is kept constant throughout the process. When the temperature of the gas is kept constant, while the pressure and volume change, Boyle's Law (page 10) applies and we have simply

$$PV = C = \text{a constant.} \quad (23)$$

This is the equation of a curve which is known in analytic geometry as a rectangular hyperbola. It is the special case of the general equation $PV^n = \text{constant}$ (22), in which the index $n = 1$. If various values are substituted for V in equation (23) and the values found together with the corresponding calculated

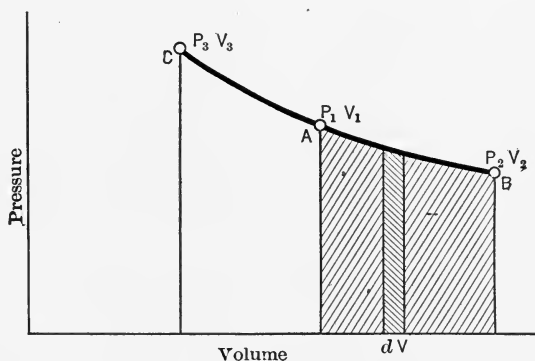


FIG. 7. — Work done by Isothermal Expansion and Compression.

values of P are plotted, a curve like the one in Fig. 7 is obtained. An equation for the work done in the isothermal expansion of a gas will now be determined, starting with the initial condition of pressure and volume represented by P_1 and V_1 . The external work performed is shown graphically by the shaded area under the curve between A and B. Two vertical lines close together in the figure are the limits of a narrow closely shaded area and indicate an infinitesimal volume change dV , so small that the pressure may be assumed constant for the interval. Work done during this **small change of volume** is, then,

$$dW = P dV,$$

and for a finite change of volume of any size as from V_1 to V_2 the work done, W (foot-pounds), is

$$W = \int_{V_1}^{V_2} P dV. \quad (24)$$

For integration of this form it is necessary to substitute P in terms of V . Assume that P and V are values of pressure and volume for any point on the curve of expansion of a gas of which the equation is

$$PV = C \quad (\text{see equation 23}).$$

Then

$$P = \frac{C}{V}.$$

Substituting this value of P in equation (24), we have

$$\begin{aligned} W &= \int_{V_1}^{V_2} \frac{C}{V} dV = C \int_{V_1}^{V_2} \frac{dV}{V}. \\ W &= C (\log_e V_2 - \log_e V_1). \end{aligned} \quad (25)$$

Since the initial conditions of the gas are P_1 and V_1 , we have

$$PV = C = P_1V_1,$$

and substituting this value of C in equation (25), we obtain

$$\begin{aligned} W &= P_1V_1 (\log_e V_2 - \log_e V_1) \\ \text{or} \quad W &= P_1V_1 \log_e \frac{V_2}{V_1} \quad (\text{in foot-pounds}). \end{aligned} \quad (26)$$

The above equation is quite general in application and can be used for any values of volume represented by V_1 and V_2 .

Units of mass do not enter. For the work done by a certain mass of gas under the same conditions we could write, since

$$\begin{aligned} P_1V_1 &= MRT \quad (\text{in foot-pounds}) \quad \text{and} \quad \frac{V_2}{V_1} = \frac{P_1}{P_2}, \\ W &= MRT \log_e \frac{V_2}{V_1} = MRT \log_e \frac{P_1}{P_2} \quad (\text{in foot-pounds}). \end{aligned} \quad (27)$$

Often the ratio $\frac{V_2}{V_1}$ is called **the ratio of expansion** and is rep-

resented by r . Making this substitution we have, in foot-pounds,

$$W = MRT \log_e r. \quad (28)$$

These equations refer to an expansion from P_1V_1 to P_2V_2 . If on the other hand, we wanted the work done in a **compression** from P_1V_1 to P_3V_3 the curve of compression would be from A to C and the area under it would be its graphical representation. Equations (26), (27) and (28) would represent the work done the same as for expansion except that the expression would have a **negative** value; that is, work is to be done **upon** the gas to decrease its volume.

The isothermal expansion or compression of a "perfect" gas causes no change in its stock of internal energy since T is constant (see page 20). During such an expansion the gas must **take in an amount of heat just equal to the work it does**, and conversely during such a compression it must **reject an amount of heat just equal to the work spent upon it**. This quantity of heat H (in B.t.u.) is, from equation (27),

$$H = \frac{MRT}{778} \log_e \frac{V_2}{V_1} \text{ (in B.t.u.)}. \quad (29)$$

The following problem shows the application of the foregoing theory to isothermal expansions:

Air having a pressure of 100 pounds per square inch absolute and a volume of 1 cubic foot expands isothermally to a volume of 4 cubic feet. Find

- (a) External work of the expansion;
- (b) Heat required to produce the expansion;
- (c) Pressure at end of expansion.

Solution. (a) Since expansion is isothermal,

$$\begin{aligned} \text{External work, } W &= P_1V_1 \log_e^* \frac{V_2}{V_1} \\ &= 100 \times 144 \times 1 \times 1.3848 \\ &= 19,941 \text{ foot-pounds.} \end{aligned}$$

* $2.3 \times \log$ base 10 = \log base e . Tables of natural logarithms are given on pages 195-196.

(b) Since the heat added equals increase in internal energy plus external work, and since the temperature remains constant (requiring therefore no heat to increase the internal energy), the internal energy equals zero and the heat added equals the work done.

$$\text{Then} \quad \text{Heat added} = \text{external work} = \frac{19,941}{778} = 25.6 \text{ B.t.u.}$$

$$\begin{aligned} \text{(c) Since} \quad P_1 V_1 &= P_2 V_2, \\ \text{then} \quad 100 \times 1 &= P_2 \times 4, \\ P_2 &= 25 \text{ pounds per square inch absolute.} \end{aligned}$$

If a gas expands and does external work without receiving a supply of heat from an external source, it must derive the amount of heat needed to do the work from its own stock of internal energy. This process is then necessarily accompanied by a lowering of **temperature** and the expansion obviously is not isothermal.

Adiabatic Expansion and Compression. Another most important mode of expansion and compression from the viewpoint of the engineer is that in which the working substance neither receives nor rejects heat as it expands or is compressed and is called **adiabatic**. A curve which shows the relation of pressures to volumes in such a process is called an **adiabatic line** (see **Fig. 8**). In any adiabatic process the substance is neither gaining nor losing heat by **conduction** or **radiation** or internal chemical action. Hence the work which a gas does in such an expansion is all done at the expense of its stock of internal energy, and the work which is done upon a gas in such a compression all goes to increase its stock of internal energy. We could secure ideally adiabatic action if we had a gas expanding, or being compressed, in a cylinder which in all parts was a perfect non-conductor of heat. The compression of gas in a cylinder is approximately adiabatic when the process is very rapidly performed, but when done so slowly that the heat has time to be dissipated by conduction the compression is more nearly isothermal. **Fig. 8** shows on a pressure-volume diagram the

difference between an isothermal and an adiabatic for expansion or compression from an initial condition P_1V_1 at A to final conditions at B and C for expansions, and at D and E for compressions.

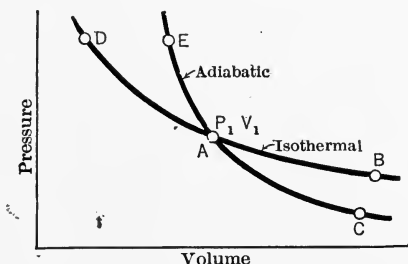


FIG. 8. — Isothermal and Adiabatic Expansion Lines.

Derivation of Equation for Adiabatic Expansion (or Compression). In order to derive the pressure-volume relation for a gas expanding adiabatically, consider the fundamental equation (page 18),

Heat added = increase in internal energy + external work or

$$H = MK_v (T_2 - T_1) + P dV \text{ (foot-pounds)}, \quad (30)$$

where K_v is the specific heat in foot-pound units, i.e., 778 C_v . Now in adiabatic expansion no heat is added or taken away from the gas by conduction or radiation, and, therefore, the left hand member of the above equation becomes zero. Furthermore, since the **combination law** (page 17) of Boyle and Charles can always be applied to perfect gases, the following simultaneous equations may be written:

$$0 = MK_v dT + P dV, \quad (31)$$

$$PV = MRT. \quad (32)$$

When P , V and T vary, as they do in adiabatic expansion, equation (32) may be written as follows:

$$P dV + V dP = MR dT, \quad (33)$$

and

$$dT = \frac{P dV + V dP}{MR}.$$

Substituting the value of dT in (31), we have

$$P dV + MK_v \frac{P dV + V dP}{MR} = 0 \quad (34)$$

$$RP dV + K_v P dV + K_v V dP = 0.$$

To separate the variables divide by PV :

$$R \frac{dV}{V} + K_v \frac{dV}{V} + K_v \frac{dP}{P} = 0. \quad (35)$$

Collecting terms,

$$(R + K_v) \frac{dV}{V} + K_v \frac{dP}{P} = 0.$$

Integrating,

$$(R + K_v) \log V + K_v \log P = \text{a constant} = c. \quad (36)$$

$$\frac{R + K_v}{K_v} \log V + \log P = c.$$

$$\log PV^{\frac{R + K_v}{K_v}} = c. \quad (37)$$

Since from equation (19),

$$R + K_v = K_p,$$

where K_p and K_v are respectively the specific heats at constant pressure and at constant volume **in foot-pound units**.

Equation (37) becomes then

$$\log PV^{\frac{K_p}{K_v}} = c,$$

and from equation (20),

$$\frac{C_p}{C_v} = \gamma \text{ and, therefore, } \frac{K_p}{K_v} = \gamma,$$

we may write

$$PV^\gamma = \text{a constant}. \quad (38)$$

Following the method used for obtaining an expression for the work done in isothermal expansion (equation 26), we can write again for the work done, W (in foot-pounds), for a change of volume from V_1 to V_2 ,

$$W = \int_{V_1}^{V_2} P dV. \quad (39)$$

Again we shall substitute, for purposes of integration, P in terms of V as outlined below. In the **general** expression $PV^n = c$, a constant (see equation 22), where P and V are values of pressure and volume for any point on the curve of expansion of a gas of which the initial condition is given by the symbols P_1 and V_1 , we can then write,

$$P = \frac{c}{V^n}. \quad (40)$$

And substituting (40) in (39),

$$W = \int_{V_1}^{V_2} \frac{c}{V^n} dV = c \int_{V_1}^{V_2} \frac{dV}{V^n}. \quad (41)$$

$$\begin{aligned} &= c \left[\frac{V^{-n+1}}{-n+1} \right]_{V_1}^{V_2} \\ &= c \left[\frac{V_2^{1-n} - V_1^{1-n}}{1-n} \right]. \end{aligned} \quad (42)$$

Since $PV^n = c = P_1V_1^n = P_2V_2^n$, we can substitute for c in (42) the values corresponding to the subscripts of V as follows:

$$W = \frac{P_2V_2^nV_2^{1-n} - P_1V_1^nV_1^{1-n}}{1-n},$$

$$W = \frac{P_2V_2 - P_1V_1}{1-n},$$

or

$$W = \frac{P_1V_1 - P_2V_2}{n-1} \text{ (foot-pounds)}. \quad (43)$$

Since

$$PV = MRT,$$

$$W = \frac{MR(T_1 - T_2)}{n-1} \text{ (foot-pounds)}. \quad (44)$$

Equations (43) and (44) apply to any gas undergoing expansion or compression according to $PV^n = \text{a constant}$. In the case of adiabatic expansion of a perfect gas $n = \gamma$ (see equation 38).

Change of Internal Energy During Adiabatic Processes. Since in adiabatic expansion no heat is conducted to or away from the gas, the work is done at the expense of the internal energy and, therefore, the latter decreases by an amount equiva-

lent to the amount of work performed. This loss in internal energy is readily computed by equation (43) or (44). The result must be divided by 778 in order to be in B.t.u.

During **adiabatic compression** the reverse occurs, i.e., there is a gain in internal energy and the same formulas apply, the result coming out negative, because work has been done on the gas.

Relation between Volume, Pressure and Temperature in Adiabatic Expansion of a Perfect Gas. Since P , V and T vary during adiabatic expansion, it will be necessary to develop formulas for obtaining these various quantities. It will be remembered that equation (10), page 15, applies to perfect gases at all times. Therefore, in the case of **adiabatic expansion or compression** we can write the two simultaneous equations:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}. \quad (A)$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma. \quad (B)$$

By means of these two equations we can find the final conditions of pressure, volume and temperature, having given two initial conditions and one final condition.

For instance, having given $V_1 V_2$ and T_1 , to find T_2 , divide (A) by (B), member for member. Then

$$\frac{V_1}{T_1 V_1^\gamma} = \frac{V_2}{T_2 V_2^\gamma},$$

$$\frac{T_2}{T_1} = \frac{V_2^{1-\gamma}}{V_1^{1-\gamma}},$$

$$T_2 = T_1 \left(\frac{V_2}{V_1} \right)^{1-\gamma},$$

or
$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1}. \quad (45)$$

In like manner the following formulas can be obtained:

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}, \quad (46)$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma, \quad (47)$$

$$P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}}, \quad (48)$$

$$V_2 = V_1 \left(\frac{P_1}{P_2} \right)^{\frac{1}{\gamma}}, \quad (49)$$

$$V_2 = V_1 \left(\frac{T_1}{T_2} \right)^{\frac{1}{\gamma-1}}. \quad (50)$$

It should be noted that the above formulas can be used for **any expansion of a perfect gas** following $PV^n = \text{a constant}$, provided γ in the formulas is replaced by n .

It is also to be noted that these equations can be used for any system of units so long as the same system of units is employed throughout an equation.

The application of these equations to a practical example will now be shown by way of illustration. Take the case of a quantity of pure air in a cylinder at a temperature of 60°F . ($T_1 = 460 + 60 = 520$ degrees absolute) which is **suddenly** (adiabatically) compressed to half its original volume. Then $\frac{V_1}{V_2} = \frac{2}{1}$, and taking γ from the table on page 23 as 1.40, the temperature immediately after compression is completed, T_2 is calculated by equation (45) as follows:

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = 520 \left(\frac{2}{1} \right)^{1.40-1} = 520 \times 2^{0.40} = 688^\circ \text{ absolute},$$

or t_2 in ordinary Fahrenheit is $688 - 460$ or 228 degrees.

The work done in adiabatic compression of one **pound** of this air is calculated by equation (44):

$$W = \frac{MR(T_1 - T_2)}{\gamma - 1} = \frac{53.3(520 - 688)}{1.40 - 1} = \frac{53.3(-168)}{0.40} = -22,386$$

foot-pounds per **pound** of air compressed. The negative sign means that work has been done **on** the gas. If the sign had

been positive it would have indicated an expansion. As the result of this compression the internal energy of the gas has been increased by $\frac{22,386}{778}$ B.t.u., but if the cylinder is a conductor of heat, as in practice it always is, the whole of this heat will become dissipated in time by conduction to surrounding air and other bodies, and the internal energy will gradually return to its original value as the temperature of the gas comes back to the initial temperature of 60° F.

During the compression the pressure rises according to equation (38):

$$PV^{\gamma} = \text{constant},$$

and just at the end the value is greater than the original pressure by the ratio of r^{γ} to 1,* or $2^{1.40}$ to 1, or 2.65 to 1. If, as before, we now assume the temperature drops gradually to the initial temperature before compression (60° F.) without changing the volume, the pressure will fall with the temperature until it has at 60 degrees a value only twice as great as the original pressure. In other words, after cooling the pressure becomes inversely proportional to the change in volume produced by the compression.

There are many cases of expansions which are neither adiabatic nor isothermal and which are not straight lines on P - V diagrams. It will be observed from the equations in the discussion of the internal work done by an expanding gas and for the change of internal energy, that if in the general equation $PV^n = \text{a constant}$ the exponent or index n is less than γ , the work done is greater than the loss in internal energy. In other words for such a case, the expansion lies between an adiabatic and isothermal and the gas must be **taking in** heat as it expands. On the other hand, if n is greater than γ the work done is less than the loss of internal energy.

* Remember r is called the ratio of expansion and is $\frac{V_2}{V_1}$. (See equation (28), page 34).

PROBLEMS

1. How many foot-pounds of work are done by 2 lbs. of air in expanding to double its volume at a constant temperature of 100°F .?

Ans. 41,400 ft.-lbs.

2. Three pounds of air are to be compressed from a volume of 2 to 1 cu. ft. at a constant temperature of 60°F . How many B.t.u. of heat must be rejected from the air?

Ans. 74.2 B.t.u.

3. An air compressor has a cylinder volume of 2 cu. ft. If it takes air at 15 lbs. per sq. in. absolute and 70°F . and compresses it isothermally to 100 lbs. per sq. in. absolute, find

(a) Pounds of air in cylinder at beginning of compression stroke.

Ans. 0.15 lb.

(b) The final volume of the compressed air. *Ans.* 0.30 cu. ft.

(c) The foot-pounds of work done upon the gas during compression.

Ans. 8200 ft.-lbs.

(d) The B.t.u. absorbed by the air in increasing the internal energy.

Ans. 0.

(e) The B.t.u. to be abstracted from the cylinder.

Ans. 10.46 B.t.u.

4. Air at 100 lbs. per sq. in. absolute and a volume of 2 cu. ft. expands along an $n = 1$ curve to 25 lbs. per sq. in. absolute pressure. Find

(a) Work done by the expansion. *Ans.* 39,900 ft.-lbs.

(b) Heat to be supplied. *Ans.* 51.3 B.t.u.

5. A quantity of air at 100 lbs. per sq. in. absolute pressure has a temperature of 80°F . It expands isothermally to a pressure of 25 lbs. per sq. in. absolute when it has a volume of 4 cu. ft. Find (1) the mass of air present, (2) Work of the expansion in foot-pounds, (3) Heat required in B.t.u.

Ans. (1) 0.50 lb.

(2) 19,950 ft.-lbs.

(3) 25.6 B.t.u.

6. Air at 100 lbs. per sq. in. absolute pressure and 2 cu. ft. expands to 25 lbs. per sq. in. absolute adiabatically. What is the final volume?

Ans. 5.4 cu. ft.

7. One cubic foot of air at 60°F . and a pressure of 15 lbs. per sq. in. absolute is compressed without loss or addition of heat to 100 lbs. per sq. in. absolute pressure. Find the final temperature and volume.

Ans. 891°F . absolute; 0.257 cu. ft.

8. Two pounds of air are expanded from a temperature of 300°F . to 200°F . adiabatically. How many foot-pounds of work are developed?

Ans. 26,300 ft.-lbs.

9. A quantity of air having a volume of 1 cu. ft. at 60°F . under a pressure of 100 lbs. per sq. in. absolute is expanded to 5 cu. ft. adiabatically.

Find the pounds of air present, the final temperature of the air and how much work will be done during this expansion. *Ans.* 0.52 lb.

273° F. absolute.

16,900 ft.-lbs.

10. Data same as Problem 3 but the compression is to be adiabatic. Find

(a) The final volume of the compressed air.

Ans. 0.516 cu. ft.

(b) The final temperature of the compressed air.

Ans. 911° F. absolute.

(c) The foot-pounds of work to compress this air.

Ans. 7510 ft.-lbs.

(d) The B.t.u. absorbed by the air in increasing the internal energy.

Ans. 9.65 B.t.u.

(e) The B.t.u. to be abstracted from the gas. *Ans.* 0 B.t.u.

11. A pound of air at 32° F. under atmospheric pressure is compressed to 4 atmospheres (absolute). What will be the final volume and the work of compression if the compression is (a) isothermal, (b) adiabatic?

Ans. (a) $V_2 = 3.09$ cu. ft.; work = 33,800 ft.-lbs.

(b) $V_2 = 4.60$ cu. ft.; work = 31,500 ft.-lbs.

CHAPTER IV

CYCLES OF HEAT ENGINES

WHEN a gas or vapor undergoes a series of processes in which there is an interchange of heat quantities and is finally brought back to the condition, as regards its physical properties, which it had initially, the gas is said to have gone through or performed a **cycle**.^{*} The most important cycle with which we have to deal is the Carnot cycle, because it is typical of the maximum efficiency obtainable.

Carnot Cycle. Very important conclusions regarding theoretically perfect heat engines are now to be drawn from the consideration of the action of an ideal engine in which the working substance is a perfect gas which is made to go through a cycle of changes involving both isothermal and adiabatic expansions and compressions. This ideal cycle of operations was invented and first explained in 1824 by Carnot, a French engineer, and gave us really the first theoretical basis for comparing heat engines with an ideally perfect engine. For explaining this Carnot cycle assume a piston and cylinder as shown in **Fig. 9**, composed of perfectly non-conducting material, except the **cylinder-head** (left-hand end of the cylinder) which is a good conductor of heat. The space in the cylinder between the piston and the cylinder head is occupied by the working substance, which we shall assume to be a perfect gas. There is provided a **hot body H** of unlimited heat capacity, always kept at a temperature T_1 , also a perfectly **non-conducting cover N** and a **refrigerating or cold body R** of unlimited heat receiving capacity, which is kept at a constant temperature T_2 (lower

^{*} A thermodynamic machine performing a cycle in which heat is changed into work is called a **heat engine**, and one performing a cycle in which heat is transferred from a medium at a low temperature to one at a higher temperature is called a **refrigerating machine**.

than T_1). It is arranged that **H**, **N** or **R** can be applied, as required, to the cylinder head. Assume that there is a **charge** of one pound of gas in the cylinder between the piston and the cylinder head, which at the beginning of the cycle, with the piston in the position shown, is at the temperature T_1 , has a volume V_a and has a pressure P_a . The subscripts attached to the

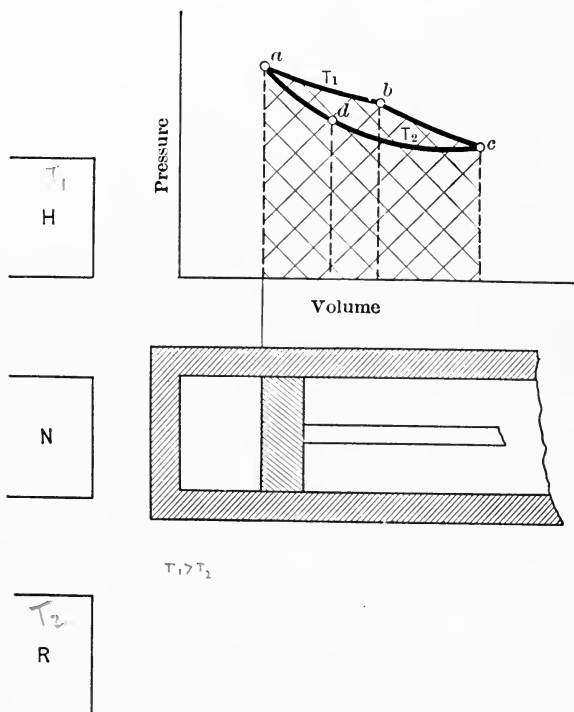


FIG. 9. — Apparatus and Diagram Illustrating a Reversible (Carnot) Cycle.

letters V and P refer to points on the pressure-volume diagram shown in the figure. This diagram shows, by curves connecting the points **a**, **b**, **c** and **d**, the four steps in the cycle.

The operation of this cycle will be described in four parts as follows: (1) Apply the hot body or heater **H** to the cylinder head* at the left-hand side of the figure. The addition of heat

* It will be remembered that the head of this cylinder is a perfect conductor of heat.

to the gas will cause it to expand **isothermally** because the temperature will be maintained constant during the process at T_1 . The pressure drops slightly to P_b when the volume becomes V_b . During this expansion external work has been done in advancing the piston and the heat equivalent of this work has been obtained from the hot body **H**.

(2) Take away the hot body **H** and at the same time attach to the cylinder head the **non-conducting cover N**. During this time the piston has continued to advance toward the right, doing work without receiving any heat from an external source so that the expansion of the gas in this step has been done at the expense of the stock of internal energy in the gas. The temperature has continued to drop * in proportion to the loss of heat to the value T_2 . Pressure is then P_c and the volume is V_c .

(3) Take away the non-conductor **N** and apply the refrigerator **R**. Then force the piston back into the cylinder. The gas will be compressed **isothermally** at the temperature T_2 . In this compression, work is being done on the gas, and heat is developed, but all of it goes into the refrigerator **R**, in which the temperature is always maintained constant at T_2 . This compression is continued up to a point **d** in the diagram, so selected that a further compression (adiabatic) in the next (fourth) stage will cause the volume, pressure and temperature to reach their initial values as at the beginning of the cycle.†

(4) Take away the refrigerator **R** and apply the non-conducting cover **N**. Then continue the compression of the gas without the addition of any heat. It will be **adiabatic**. The pressure and the temperature will rise and, if the point **d** has been properly selected, when the pressure has been brought back to its initial value P_a the temperature will also have risen to its initial value T_1 . The cycle is thus finished and the gas is ready for a

* A pressure-volume diagram of a perfect gas does not show, graphically, changes of temperature.

† Briefly the **third** stage of the cycle must be stopped when a point **d** is reached, so located that an adiabatic curve ($PV^\gamma = \text{constant}$) drawn from it will pass through the "initial" point **a**.

repetition of the same series of processes comprising the cycle.

To define the Carnot cycle completely we must determine how to locate algebraically the proper place to stop the third step (the location of **d**). During the second step (**adiabatic expansion** from **b** to **c**) we can write, by applying equation (45), the following temperature and volume relations:

$$\frac{T_1}{T_2} = \left[\frac{V_c}{V_b} \right]^{\gamma-1};$$

also for the **adiabatic compression** in the **fourth** step we can state similarly,

$$\frac{T_1}{T_2} = \left[\frac{V_d}{V_a} \right]^{\gamma-1}.$$

Hence,

$$\left[\frac{V_c}{V_b} \right]^{\gamma-1} = \left[\frac{V_d}{V_a} \right]^{\gamma-1}.$$

Simplifying and transposing, we have

$$\frac{V_b}{V_a} = \frac{V_c}{V_d}. \quad (51)$$

Observe that $\frac{V_b}{V_a}$ is the ratio of expansion r (page 33) for the **isothermal** expansion in the first step of the cycle. This has been shown to be equal to $\frac{V_c}{V_d}$ in the isothermal compression in the third step in order that the adiabatic compression occurring in the fourth step shall complete the cycle.

A summary of the heat changes to and from the working gas (per pound) in the **four steps** of the Carnot cycle is as follows:

- (ab). Heat **taken in** from hot body $= RT_1 \log_e \frac{V_b}{V_a}$ (by equation (28), in foot-pounds). (52)
- (bc). No heat taken in or rejected.

(cd). Heat rejected to refrigerator = $RT_2 \log_e \frac{V_d}{V_c}$
 or $-RT_2 \log_e \frac{V_c}{V_d}$ * (by equation (28), in foot-pounds). (53)

(da). No heat taken in or rejected.

Hence, the net amount of work done, **W**, by the gas in this cycle, being the mechanical equivalent (foot-pounds), of the **excess** of heat taken in over that rejected, is the **algebraic sum** of (52) and (53):

$$W = R \left(T_1 \log_e \frac{V_b}{V_a} - T_2 \log_e \frac{V_c}{V_d} \right) = R (T_1 - T_2) \log_e \frac{V_b}{V_a}. \quad (54)$$

If the curves in **Fig. 9** are accurately plotted to scales of pressure and volume, then the work in foot-pounds as calculated, from the measured area included in the *P-V* diagram will be found to agree exactly with the result given by equation (54) above.

Efficiency of Carnot's Cycle. The thermal or heat efficiency of a cycle is usually defined as the ratio of

$$\frac{\text{Heat equivalent of work done.}}{\text{Heat taken in}}$$

The heat equivalent of **work done** is, by equation (54),

$$R(T_1 - T_2) \log_e \frac{V_b}{V_a},$$

and the **heat taken in** is, by equation (52),

$$RT_1 \log_e \frac{V_b}{V_a}.$$

The ratio above representing the efficiency *E* is

$$E = \frac{R (T_1 - T_2) \log_e \frac{V_b}{V_a}}{RT_1 \log_e \frac{V_b}{V_a}} = \frac{T_1 - T_2}{T_1}. \quad (55)$$

$$* \text{Log } \frac{V_d}{V_c} = - \log \frac{V_c}{V_d}.$$

This efficiency represents the proportion of the total amount of heat given to the gas employed in a Carnot cycle, which the engine converts into work. At the temperature of the hot body **H**, the engine takes in an amount of heat proportional to the absolute temperature of this body or T_1 , and in the course of the cycle rejects to the refrigerator **R** an amount of heat proportional to its absolute temperature T_2 . The range of temperature for the cycle is, therefore, between T_1 and T_2 . In the lowering of temperature corresponding to this range the engine converts into work that part of the heat taken in that is represented by equation (55). Observe the conditions affecting the maximum efficiency of this cycle. For a given heat supply proportional to T_1 the only way to increase efficiency is by reducing T_2 , so that the smaller T_2 is the greater the temperature range ($T_1 - T_2$) becomes and the higher the efficiency will be.

Reversible Cycles. A heat engine which is capable of discharging to the "source of heat" when running in the reverse direction from that of its normal cycle the same quantity of heat that it would take from this source when it is running direct and doing work is said to operate with its cycle reversed, or, in other words, the engine is reversible. A reversible heat engine then is one which, if made to follow its indicator diagram in the reverse direction, will require the same horse power to drive it as a refrigerating machine as the engine will deliver when running direct, assuming that the quantity of heat used is the same in the two cases. An engine following Carnot's cycle is, for example, a reversible engine. The thermodynamic idea of **reversibility** in engines is of very great serviceableness because it will be shown that no heat engine can be more efficient than a reversible engine when both work between the same limits of temperature; that is, when both engines take in the same amount of heat at the same higher temperature and reject the same amount at the same lower temperature.

Carnot's Principle. It was first proved conclusively by Carnot that no other heat engine can be more efficient than a reversible engine when both work between the same temperature.

limits. To illustrate this principle, assume that there are two engines **A** and **B**. Of these let us say **A** is reversible and **B** is not. In their operation both take heat from a hot body or heater **H** and reject heat to a refrigerator or cold body **R**. Let Q_H be the quantity of heat which the reversible engine **A** takes in from the hot body **H** for each unit of work performed, and let Q_R be the quantity of heat per unit of work which it discharges to the refrigerator **R**.

For the purpose of this discussion, assume that the non-reversible engine **B** is more efficient than the reversible engine **A**. Under these circumstances it is obvious that the engine **B** will take in less heat than **A** and it will reject correspondingly less heat to **R** per unit of work performed. The heat taken in by the non-reversible engine **B** from the hot body **H** we shall designate then by a quantity less than Q_H or $Q_H - X$ and the heat rejected by **B** to the refrigerator **R** by $Q_R - X$. Now if the non-reversible engine **B** is working direct (when converting heat into work) and is made to drive the reversible engine **A** according to its reverse cycle (when converting work into heat), then for every unit of work done by the engine **B** in driving the reversible engine **A**, the quantity of heat mentioned above, that is, $Q_H - X$, would be taken from the hot body **H** by the non-reversible engine **B** and, similarly, the quantity of heat represented by Q_H would be returned to the hot body **H** by the reverse action of the cycle of operations performed by **A**. This follows because the engine **A** is reversible and it returns, therefore, to **H**, when operating on the reverse cycle, the same amount of heat as it would take in from **H** when working on its direct cycle. By this arrangement the hot body **H** would be continually receiving heat, in the amount represented by X for each unit of work performed. At the same time the non-reversible engine **B** discharges to the refrigerator **R** a quantity of heat represented by $Q_R - X$, while the reversible engine **A** removes from the refrigerator **R** a quantity represented by Q_R . As a result of this last operation the cold body will be losing continually per unit of work performed a quantity of heat equal to X . The combined per-

formances of the two engines, one working direct as a normal heat engine and the other, according to its reverse cycle, as a compressor or what might be called a "heat pump," gives a constant removal of heat from the refrigerator **R** to the hot body **H**, and as a result a degree of **infinite coldness** must be finally produced in the refrigerator.

If we assume that there is no mechanical friction, this combined machine, consisting of a normal heat engine and compressor, will require no power from outside the system. For this reason the assumption that the non-reversible engine **B** can be more efficient than the reversible engine **A** has brought us to a result which is impossible from the standpoint of experience as embodied in the statement of the "Second Law of Thermodynamics" (see page 3); that is, it is impossible to have a self-acting engine capable of transferring heat, infinite in quantity, from a cold body to a hot body. We should, therefore, conclude that no non-reversible engine, as **B** for example, can be more efficient than a reversible engine **A** when both engines operate between the same temperature limits. More briefly, when the source of heat and the cold receiver are the same for both a reversible heat engine and any other engine, then the reversible engine must have a higher possible efficiency; and if both engines are reversible it follows that neither can be more efficient than the other.

Perfection in a Heat Engine. A reversible engine is perfect from the viewpoint of efficiency; that is, its efficiency is the best obtainable. No other engine than a reversible engine which takes in and discharges heat at identical temperatures will transform into work a greater part of the heat which it takes in. Finally, it should be stated as regards this efficiency that the nature of the substance being expanded or compressed has absolutely no relation to the thermal efficiency as outlined above.

Reversed Carnot's Cycle. If an engine operating on Carnot's cycle is reversed in its action so that the same indicator diagram shown in **Fig. 9** would be traced in the opposite direction, the reversed cycle, when beginning as before at **a** with a per-

fect gas at the temperature T_1 , will consist of the following stages:

(1) When the non-conductor **N** is applied and the piston is advanced toward the right by the source of power performing the reversed cycle, the gas will expand, tracing the adiabatic curve **ad**, with constant lowering of temperature which at the point **d** will be T_2 .

(2) When the non-conductor **N** is now removed, the refrigerator **R** is applied, and the piston continues on its outward stroke. The gas will expand isothermally at the constant temperature T_2 , tracing the curve **dc**. During this stage the gas is taking heat from the refrigerator **R**.

(3) When the refrigerator **R** is removed and the non-conductor **N** is again applied, which will be on the back stroke of the engine, the gas will be compressed, and on the indicator diagram another adiabatic curve **cb** will be traced. At the point **b** the temperature will be obviously T_1 .

(4) When the non-conductor **N** is removed and the hot body **H** is again applied, with the compression continuing along the isothermal curve **ba**, heat will be discharged to the hot body **H**, while the temperature is maintained constant at T_1 . The cycle has now been traced in a reverse direction from the beginning back to the starting point at **a**, and is now complete. During this process no work has been done, but on the contrary an amount of work represented by the area of the indicator diagram, equivalent in foot-pounds to

$$R \log_e \frac{V_b}{V_a} (T_1 - T_2) \text{ (see equation (54), page 48),}$$

has been converted into heat. First, heat was **taken from** the refrigerator **R**, represented in amount by

$$RT_2 \log_e \frac{V_c}{V_d},$$

and second, heat was **rejected to** the hot body **H** in the amount

$$RT_1 \log_e \frac{V_a}{V_b} \quad \text{or} \quad -RT_1 \log_e \frac{V_b}{V_a}.$$

As in direct operation of Carnot's cycle no heat is given or lost in the first and third stages outlined above. The algebraic sum of these two quantities, remembering that $\frac{V_b}{V_a} = \frac{V_c}{V_d}$, gives the net amount of work done, W , on the gas, and, therefore, the net amount of heat (foot-pound units) transferred from the cold body **R** to the hot body **H** or,

$$W = RT_2 \log_e \frac{V_b}{V_a} - RT_1 \log_e \frac{V_b}{V_a} = -R \log_e \frac{V_b}{V_a} (T_1 - T_2). \quad (56)$$

Since the result is the same as given by equation (54), although opposite in sign on account of being work of compression, it will be observed that in the reverse cycle the same amount of heat is given to the hot body **H** as was taken from it in the direct operation of the same cycle, and that the same amount of heat is now taken from the refrigerator **R** as was in the other case given to it.

Conclusions from the Above Discussion. In the explanation that has preceded of the performance of heat engines a perfect gas took in heat at the temperature of the source of supply of heat and discharged heat at the temperature of the refrigerator, or receiver of heat, the changes of temperature occurring as the result of adiabatic expansion or adiabatic compression. For a perfect gas we have determined, as in equation (55), that the efficiency of the heat engine was $(T_1 - T_2) \div T_1$. It has also been shown that Carnot's cycle was reversible, and that according to the "Second Law of Thermodynamics" no heat engine can have a higher efficiency than such a reversible engine when taking in and discharging heat at the same two temperatures T_1 and T_2 . Finally, we are brought to the important conclusion that all reversible heat engines receiving and discharging heat at the same temperatures T_1 and T_2 are of equal efficiency, and that this efficiency,

$$E = \frac{T_1 - T_2}{T_1}, \quad (57)$$

having been determined for one reversible engine is also the efficiency of any other reversible engine, and is the maximum efficiency attainable with any engine.

It should be observed in connection with the statement of an equation of efficiency like (57) above that it is impossible to utilize the whole of any supply of heat for conversion into work because it is impossible to reach the absolute zero of temperature, or in other words to make T_2 in this equation practically zero. Considering only practical conditions, therefore, we may say that with given limits of temperatures T_1 and T_2 it is necessary for the attainment of the greatest efficiency that no heat shall be taken in by an engine except during the isothermal at the highest temperature and that no heat shall be rejected except at the isothermal at the lowest temperature T_2 .

The following problem shows the application of the theory of Carnot's cycle:

In Carnot's cycle the gas has an initial condition of 100 pounds per square inch absolute pressure, volume of 1 cubic foot and temperature of 300° F. The volume at the end of isothermal expansion is 2 cubic feet. Exhaust temperature is 60° F. Find

- (a) Heat supplied to the cycle;
- (b) Efficiency of the cycle;
- (c) Net work of the cycle.

Solution. The heat supplied equals

$$100 \times 144 \times 1 \times \log_e \frac{2}{1} = 9970 \text{ foot-pounds or } 12.8 \text{ B.t.u.}$$

The efficiency is

$$\frac{(300 + 460) - (60 + 460)}{300 + 460} = 0.316 \text{ or } 31.6 \text{ per cent.}$$

The net work of the cycle is

$$9970 \text{ foot-pounds} \times 0.316 = 3150 \text{ foot-pounds.}$$

Regenerative Air Engines. The previous discussion has dealt entirely with ideal engines following what is known as Carnot's cycle. Such an engine has never been built. Another engine having theoretically a reversible cycle has, therefore, the same

practical application and will be next described. This engine, known as Stirling's, consists of two cylinders side by side, one of which is heated and the other cooled. It performs its cycle according to the following four stages:

(1) Air which has been previously heated in a **regenerator** * to a temperature T_1 is expanded isothermally from a volume represented by V_1 to a volume V_2 . During this stage heat is being taken in from the furnace and the piston is raised as the expansion proceeds. The heat taken in during this stage in foot-pounds per pound of air is

$$RT_1 \log_e \frac{V_b}{V_a} \text{ (see equation 52).}$$

(2) During the next stage this air which is now highly heated is made to pass through the regenerator placed between the hot and the cold cylinders, and in its passage through the regenerator it gives up heat and has its temperature reduced to T_2 , without a change of volume. The heat absorbed by the regenerator is $C_v(T_1 - T_2)$. There is, of course, a drop in pressure corresponding to the reduction in temperature, although there is no change of volume.

(3) In the cooled cylinder the air is compressed isothermally at the temperature T_2 to its original volume. The heat discharged in this stage is

$$RT_2 \log \frac{V_d}{V_c} = - RT_2 \log \frac{V_b}{V_a}.$$

(4) The air is now passed back through the regenerator from the cooled cylinder to the heated cylinder, absorbing heat from the regenerator on the way and having its temperature raised to that at the beginning of the cycle or T_1 . Heat taken in from the regenerator in this stage is $C_v(T_1 - T_2)$.

* The regenerator consisting of a series of iron plates serves as a heat accumulator. When hot gases are passed through these plates they become heated. On the other hand, when cold gases are passed through the plates the heat previously absorbed would be given up to the gas.

The thermal efficiency of the whole cycle is then

$$E = \frac{\text{Heat taken in} - \text{Heat discharged}}{\text{Heat taken in}}$$

This would be expressed by the symbols already used as

$$E = \frac{RT_1 \log_e \frac{V_b}{V_a} - RT_2 \log_e \frac{V_b}{V_a}}{RT_1 \log_e \frac{V_b}{V_a}}.$$

Canceling out the common terms, we have more simply

$$E = \frac{T_1 - T_2}{T_1}.$$

A theoretical indicator diagram from Stirling's engine is shown in **Fig. 10.**

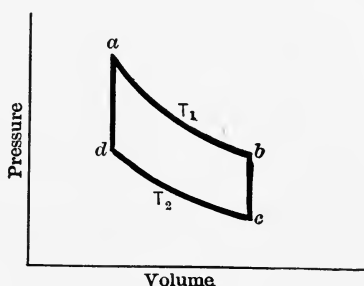


FIG. 10. — Indicator Diagram of Stirling's Air Engine.

This engine, although not commercially a success, is, however, important because it represents the only **type** besides Carnot's that is reversible. The application of regenerators in heat engines is very limited and is confined almost exclusively to engines using hot air as the working substance.

Another method of using air as the working substance in heat engines was developed by Ericsson, who used a cycle consisting of constant pressure and isothermal lines. The expansion took place at constant pressure while the air was passing through the regenerator.

The following problem shows the application of the theory to cycles other than the Carnot. (Stirling engine.)

Assume a cylinder of 1 cubic foot volume which contains air at 60° F. and 15 pounds per square inch absolute pressure and the cycle of operation to be performed as follows:

(1) The gas is compressed adiabatically until the pressure equals 100 pounds per square inch absolute.

(2) Heat is then supplied without change of volume of the gas and raises the temperature to 100° F.

(3) The gas then expands adiabatically to a volume of 1 cubic foot.

(4) Heat is then rejected without change of volume until the temperature is lowered to 60° F. The mass of air present is

$$M = \frac{15 \times 144 \times 1}{53.3 (60 + 460)} = 0.078 \text{ lb.}$$

The temperature at the end of adiabatic compression is

$$T_2 = (60 + 460) \left\{ \frac{100}{15} \right\}^{\frac{1.4-1}{1.4}} = 895^\circ \text{ F. absolute.}$$

The temperature after addition of the heat is

$$T_3 = 895 + 100 = 995^\circ \text{ F. absolute.}$$

The volume after adiabatic compression is

$$V_2 = \frac{0.078 \times 53.3 \times 895}{100 \times 144} = 0.258 \text{ cu. ft.}$$

The temperature after adiabatic expansion equals

$$T_4 = \frac{995 (0.228)^{1.4-1}}{1^{1.4-1}} = 578^\circ \text{ F. absolute.}$$

The heat supplied to the cycle is

$$0.078 \times 0.169 \times 100 = 1.32 \text{ B.t.u.}$$

The heat exhausted from the cycle is

$$0.078 \times 0.169 \times (578 - 520) = 0.76 \text{ B.t.u.}$$

The net work of the cycle is

$$1.32 - 0.76 = 0.56 \text{ B.t.u. or 436 foot-pounds.}$$

The efficiency of the cycle is

$$\frac{0.56}{1.32} = 0.42 \text{ or 42 per cent.}$$

PROBLEMS

1. A Carnot engine containing 10 lbs. of air has at the beginning of the expansion stroke a volume of 10 cu. ft. and a pressure of 200 lbs. per sq. in. absolute. The exhaust temperature is 0°F . If 10 B.t.u. of heat is added to the cycle, find

(a) Efficiency of the cycle. *Ans.* 15 per cent.

(b) Work of the cycle. *Ans.* 1160 ft.-lbs.

2. A Carnot cycle has at the beginning of the expansion stroke a pressure of 75 lbs. per sq. in. absolute, a volume of 2 cu. ft. and a temperature of 200°F . The volume at the end of isothermal expansion is 4 cu. ft. The exhaust temperature is 30°F . Find

(a) Heat added to cycle. *Ans.* 19.3 B.t.u.

(b) Efficiency of cycle. *Ans.* 25.8 per cent.

(c) Work of cycle. *Ans.* 3860 ft.-lbs.

3. A cycle made up of two isothermal and two adiabatic curves has a pressure of 100 lbs. per sq. in. absolute and a volume of 1 cu. ft. at the beginning of isothermal expansion. At the end of adiabatic expansion the pressure is 10 lbs. per sq. in. absolute and the volume is 8 cu. ft. Find

(a) Efficiency of cycle.* *Ans.* 20 per cent.

(b) Heat added to cycle. *Ans.* 28.5 B.t.u.

(c) Net work of cycle. *Ans.* 4360 ft.-lbs.

4. In a steam power plant the steam is generated at 400°F . and is exhausted at 216°F . If the heat in the steam could be transformed according to Carnot's cycle, what would be the efficiency of the plant?

Ans. 21.4 per cent.

5. In a Carnot cycle the heat is added at a temperature of 400°F . and rejected at 70°F . The working substance is 1 lb. of air which has a volume of 2 cu. ft. at the beginning and a volume of 4 cu. ft. at the end of isothermal expansion. Find

(a) Volume at end of isothermal compression. *Ans.* 6.70 cu. ft.

(b) Heat added to the cycle. *Ans.* 40.7 B.t.u.

(c) Heat rejected from cycle. *Ans.* 25.2 B.t.u.

(d) Net work of the cycle. *Ans.* 12,150 ft.-lbs.

6. Air at a pressure of 100 lbs. per sq. in. absolute, having a volume of 1 cu. ft. and a temperature of 200°F ., passes through the following operations:

1st. Heat is supplied to the gas while expansion takes place under constant pressure until the volume equals 2 cu. ft.

2nd. It then expands adiabatically to 15 lbs. per sq. in. absolute pressure.

$$* \text{ Efficiency} = \frac{P_1 V_1 - P_2 V_2}{P_1 V_1} \text{ or } \frac{T_1 - T_2}{T_1}.$$

3d. Heat is then rejected while compression takes place under constant pressure.

4th. The gas is then compressed adiabatically to its original volume of 1 cu. ft.

Find (a) Pounds of air used.

Ans. 0.41 lb.

(b) Temperature at end of constant pressure expansion.

Ans. 1320° F. absolute.

(c) Heat added to the cycle.

Ans. 64.3 B.t.u.

(d) Net work of cycle.

Ans. 21,160 ft.-lbs.

(e) Efficiency of cycle.

Ans. 42.2 per cent.

CHAPTER V

PROPERTIES OF STEAM

Steam in Heat Engines. The discussion on the preceding pages has had to do largely with the action of **perfect gases** in heat engines. Now we shall consider in this chapter a more limited field, confining ourselves exclusively to the action of water vapor or **steam** in such engines. The unusual physical properties of steam must, therefore, be explained here in considerable detail as well as the use and application of tables of steam properties, called, for short, **steam tables**. Nearly all our dealings with steam in an engineering way will have to do with its formation at **constant pressure**. This is the condition of steam formation in a power plant boiler when the engines are at work.

To make perfectly clear the process of steam formation in a boiler, assume that steam is to be made in a cylinder having a cross-sectional area of one square foot, closed at one end on which it stands. This cylinder is fitted with a frictionless piston (**Fig. 11**),* which is loaded so that it will exert a constant pressure of 100 pounds per square foot on the fluid in the cylinder below the piston. To begin the explanation of the properties of water vapor or steam, assume that there is in the bottom of the cylinder a quantity of **water**, say one pound, at just 32° F. (not ice). If, now, heat is applied to the bottom of the cylinder, it will pass through the walls of the cylinder and will enter the water where it will produce the following changes in three stages:

(1) As the water takes in heat its temperature rises until a certain temperature t is reached, at which steam begins to form. The value of t depends on the particular pressure which the

* Compare with Fig. 2, page 9.

piston and its load exerts. Until the temperature t is reached there is nothing but water below the piston.

(2) After heating the water to the temperature t corresponding to the particular pressure exerted, and then adding more heat, there is no further rise in temperature, but water vapor (steam) begins to form. With this formation of steam there is a rapid increase in volume, and the frictionless piston which is sup-

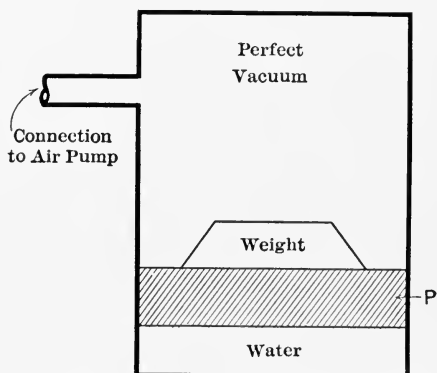


FIG. 11.—Simple Apparatus to Illustrate Pressure and Volume Relations of Steam.

posed to exert a constant pressure will be raised. The formation of steam at constant temperature and constant pressure continues throughout this stage until all the water is converted into steam. All the steam which is formed during this stage is said to be **saturated**.

(3) If, after all the water has been evaporated into steam, still more heat is added and taken in, the volume will be still further increased and there will also be an increase in temperature. The steam in this last stage is then said to be **superheated**. To make clear this distinction between saturated and superheated steam, it should be added that if at a given pressure steam exists at a temperature higher than the temperature (t) "of saturation," it is said to be superheated. The difference between saturated and superheated steam may also be expressed by saying that if **water** (at the temperature of the steam) **be**

mixed with the steam some of this water will be gradually evaporated if the steam is superheated, but not if the steam is saturated.*

The properties of saturated steam differ very much from those of the perfect gases which we have been studying, but when the steam is superheated to a high degree its properties approach very closely those of an ideally perfect gas. It is most important to remember that saturated steam at any particular pressure is always at the same temperature, while, on the other hand, superheated steam can have any temperature higher than that corresponding to saturated steam at the same pressure.

Relation of Temperature, Pressure and Volume in Saturated Steam. The important relations of temperature, pressure and volume were first determined in a remarkable series of experiments conducted by a French engineer named Regnault, and it has been on the basis of his data, first published in 1847, that even our most modern steam tables are based. Later experimenters have found, however, that these data were somewhat in error, especially for values near the dry saturated condition. These errors resulted because it was difficult in the original apparatus to obtain steam entirely free from moisture.

The pressure of saturated steam increases very rapidly as the temperature increases in the upper limits of the temperature scale. It is very interesting to examine a table of the properties of steam to observe how much more rapidly the pressure must be increased in the higher limits for a given range of temperature.

It should be observed that in most tables the pressure is almost invariably given in terms of pounds per **square inch**, while in nearly all our thermodynamic calculations the pressure must be used in pounds per **square foot**.

Heat in the Liquid (Water) (h). The essentials of the process of making steam have been above described in a general way.

* It is not unusual at all to find in practice hot water existing indefinitely in the presence of superheated steam. See Moyer's *Steam Turbines*, page 262, and *Power Plant Testing* by the same author, page 316.

The relation of this process to the **amount** of heat required will now be explained. If a pound of water which is initially at some temperature t_0 is heated at a constant pressure P (pounds per square foot) to the boiling point corresponding to this pressure and then converted into steam, heat will first be absorbed in raising the temperature of the water from t_0 to t , and then in producing vaporization. During the first stage, while the temperature is rising, the amount of heat taken in is approximately $(t - t_0)$ heat units, that is, British thermal units (B.t.u.), because the specific heat of water is approximately unity and practically constant. This number of B.t.u. multiplied by 778 gives the equivalent number of foot-pounds of work. For the purpose of stating in steam tables the amount of heat required for this heating of water, the initial temperature t_0 must be taken at some definite value; for convenience in numerical calculations and also because of long usage, the temperature 32°F. is invariably used as an arbitrary starting point for calculating the amount of heat "taken in." The symbol h (or sometimes q) is used to designate the heat required to raise one pound of water from 32°F. to the temperature at which it is vaporized into steam. In other words, "the heat of the liquid" (h) is the amount of heat in B.t.u. required to raise one pound of water from 32°F. to the boiling point.

It is obvious, therefore, that we can write the value of the heat absorbed by water in being raised to the steaming temperature (h) in B.t.u., approximately, by the formula

$$h = t - 32 \text{ (in B.t.u.)}. \quad (58)$$

More accurate values of h , taking into consideration the variation in the specific heat of water, will be found in the usual steam tables. During this first stage, before any steaming has occurred, practically all the heat applied is used to increase the stock of internal energy. The amount of external work done by the expansion of water as a liquid is practically negligible.

Latent Heat of Evaporation (L). In the second stage of the formation of steam as described, the water at the temperature t ,

corresponding to the pressure, is changed into steam at that temperature. Although there is no rise in temperature, very much heat is nevertheless required to produce this evaporation or vaporization. The heat taken in during this stage is the **latent heat of steam**. In other words, the latent heat of steam may be defined as the amount of heat which is taken in by a pound of water while it is changed into steam at constant pressure, the water having been previously heated up to the temperature at which steam forms. The symbol **L** (also sometimes **r**) is used to designate this latent heat of steam. Its value varies with the particular pressure at which steaming occurs, being somewhat smaller in value at high pressures than at low.

External Work of Evaporation (E). A part of the heat taken in during the "steaming" process is spent in doing external work.* Only a small part of the heat taken in is represented by the external work done in making the steam in the boiler, and the remainder of the latent heat (**L**) goes to increase the internal energy of the steam. The amount of heat that goes into the performing of external work is obviously equal to **P** (the pressure in pounds per square foot) times the change of volume occurring when the water is changed into steam.

Example. At the usual temperatures of the working fluid in steam engines the volume of a pound of water is about $\frac{1}{60}$ of a cubic foot. The external work, **W**, done in making one pound of steam having finally a volume of **V** (cubic feet) at a constant pressure **P** (pounds per square foot) may be written in foot-pounds:

$$W = \text{External work} = P (V - \frac{1}{60}). \quad (59)$$

This last equation can be expressed in British thermal units (B.t.u.) by dividing by 778. It is apparent also from this equa-

* The external work done in the formation of steam at constant pressure would be illustrated by the apparatus used for explaining the three stages of steam formation on page 61. It should be observed that while heat is being added merely to raise the temperature of the water there is practically no movement of the piston as there is scarcely any change in volume, but when steam is being made the volume increases rapidly and the piston will rise high above its initial position.

tion that the external work done in making steam is less at low pressure than at high,* because there is less resistance to overcome, or, in other words, P in equation (59) is less. The heat equivalent of the external work is, therefore, a smaller proportion of the heat added at low temperature than at high.

Total Heat of Steam (H). The heat added during the process represented by the first and second stages in the formation of a pound of steam, as already described, is called the total heat of saturated steam or, for short, **total heat of steam**, and is represented by the symbol H . Using the symbols already defined, we can write, per pound of steam,

$$H = h + L \text{ (B.t.u.)}. \quad (60)$$

In other words, this total heat of steam is the amount of heat required to raise one pound of water from 32°F. to the temperature of vaporization and to vaporize it at that temperature under a constant pressure.

Remembering that h for water is approximately equal to the temperature less 32 degrees corresponding to the pressure at which the steam is formed (t), we can also write approximately,

$$H = (t - 32) + L. \quad (61)$$

To illustrate that equation (61) is approximately correct, take the case of steam being formed in a boiler at an absolute pressure of 115 pounds per square inch.

From the steam tables we find that the temperature t of the steam at this pressure is 338°F. , the latent heat of vaporization is 880 B.t.u. per pound and the total heat of the steam is 1189 B.t.u. per pound. To check these values with equation (59), we have, by substituting values of L and t ,

$$H = (338 - 32) + 880 = 1186 \text{ B.t.u. per pound.}$$

* Although at the lower pressure the volume of a given weight of steam is greater than at a higher pressure, the change of pressure is relatively so much greater in the process of steam formation that the product of pressure and change of volume, $P(V_2 - V_1)$, which represents the external work done, is less for low pressure steam than for high.

When steam is condensed under constant pressure, obviously the process which we have called the "second stage" is reversed and the amount of heat equal to the latent heat of evaporation (L) is given up during the change that occurs in the transformation from steam to water.

Internal Energy of Evaporation and of Steam. It was explained in a preceding paragraph that when steam is forming not all of the heat added goes into the **internal** or "**intrinsic**" energy of the steam, but that a part of it was spent in performing external work. If, then, we represent the **internal energy** of evaporation by the symbol I_L , we can write, similarly to equation (59), in B.t.u. per pound of steam,

$$I_L = L - P \frac{(V - \frac{1}{60})^*}{778}. \quad (62)$$

This equation represents the increase in internal energy which takes place in the changing of a pound of water at the temperature t into steam at the same temperature. In all the formulas dealing with steam that we have used we adopt the state of water at 32°F. as the **arbitrary starting point** from which the taking in of heat was calculated. This same arbitrary starting point is used also in expressing the amount of internal energy in the steam. This is the excess of the heat taken in over the external work done in the process. The **total internal energy** (I_H) of a pound of saturated steam at a pressure P in pounds per square foot is equal to the total heat (H) less the heat equivalent of the external work done; thus,

$$I_H = H - P \frac{(V - \frac{1}{60})}{778}. \quad (63)$$

Such reference is made here to the internal energy of steam because it is very useful in calculating the heat taken in and rejected by steam during any stage of its expansion or com-

* It must be remembered that whenever there is a product of pressure and volume the result is in **foot-pounds**. To combine such a result with other terms in B.t.u. we must divide by 778.

pression. It is well to recall the following brief and simple statement (equation 12, page 18):

Heat taken in = increase of internal energy + external work done.

When we are dealing with a compression instead of an expansion then the last term above (external work) will be a negative value to indicate that work is done upon the steam instead of the steam doing work by expansion.

The following problem shows the calculation of internal energy and external work:

Example. A boiler is evaporating water into dry and saturated steam at a pressure of 300 pounds per square inch absolute. The feed water enters the boiler at a temperature of 145° F.

The internal energy of evaporation per pound of steam is

$$\begin{aligned} I_L &= L - \frac{P(V - \frac{1}{60})}{778} = 811.3 - \frac{300 \times 144 (1.551 - \frac{1}{60})}{778} \\ &= 811.3 - 85.3 = 726.0 \text{ B.t.u.,} \end{aligned}$$

or taken directly from saturated tables equals 726.8 B.t.u.

The total internal energy supplied above 32° F. per pound of steam is

$$I_H = H - \frac{P(V - \frac{1}{60})}{778} = 1204.1 - 85.3 = 1118.8 \text{ B.t.u.,}$$

or taken directly from saturated tables equals 1118.5 B.t.u.

External work done above 32° F. per pound of steam as calculated from steam tables is

$$H - I_H = 1204.1 - 1118.5 = 85.6 \text{ B.t.u.}$$

External work of evaporation per pound of steam as calculated from the steam tables is

$$L - I_L = 811.3 - 726.8 = 84.5 \text{ B.t.u.}$$

The external work done in raising the temperature of a pound of water from 32° F. to the boiling point is

$$85.6 - 84.5 = 1.1 \text{ B.t.u.}$$

The external work done in raising the temperature of a pound of water from 32° F. to 145° F. is

$$\frac{300 \times 144 (0.0163 - 0.01602^*)}{778} = 0.01 \text{ B.t.u.}$$

The external work done in forming the steam from water at 145° F. is, then,

$$84.5 + 1.10 - 0.01 = 85.59 \text{ B.t.u.}$$

as compared with 85.3 B.t.u. as calculated from

$$\frac{P(V - \frac{1}{60})}{778}$$

It is to be noticed that the external work done during the addition of the heat of the liquid is small as compared with other values, and for most engineering work it is customary to assume that no external work is done during the addition of heat to the water. With this assumption

$$I_H = h + L - \frac{P(V - \frac{1}{60})}{778} = H - (L - I_L).$$

Steam Formed at Constant Volume. When saturated steam is made in a boiler at **constant volume**, as, for example, when the piping connections from the boiler to the engines are closed, then no external work is done, and all the heat taken in is converted into and appears as internal energy I_H of the steam. This quantity is less than the total heat H of steam, representing its formation at constant pressure, by quantity $P(V - \frac{1}{60}) \div 778$, where P represents the absolute pressure at which the steam is formed in pounds per square foot and V is the volume of a pound of steam at this pressure in cubic feet.

Wet Steam. In all expansions studied thus far, dealing with saturated steam, it has been assumed that the steaming process was complete and that the water had been completely converted into steam. Now in actual engineering practice it is not at all unusual to have steam leaving boilers which is not perfectly

* See Table 6 in Marks and Davis' *Steam Tables and Diagrams* for volumes of water.

and completely vaporized; in other words, the boilers are supplying to the engines a sort of mixture of steam and water. This mixture we call **wet steam**. It is steam which carries actually in suspension minute particles of water, which remain thus in suspension almost indefinitely. The temperature of this wet steam is always the same as that of completely saturated steam as given in the steam tables so long as any steam remains uncondensed. The ratio of the weight of moisture or water in a pound of **wet steam** to a pound of completely saturated steam is called the **degree of wetness**; and when this ratio is expressed as a per cent, we have then what we call **percentage of moisture** or "**per cent wet**"; thus, if in a pound of wet steam there is 0.04 pound of water in suspension or entrained, we speak of the steam as being four per cent wet. Another term, called the **quality of steam**, which is usually expressed by the symbol x , is also frequently used to represent the condition of wet steam. Quality of steam may be defined as the proportion of the amount of dry or completely evaporated steam in a **pound of wet steam**. To illustrate with the example above, if there is 0.04 pound of water in a pound of wet steam; the **quality in this case would be $1 - 0.04$ or 0.96** . In this case we say then that the quality of this steam is ninety-six one-hundredths.

With this understanding of the nature of wet steam it is obvious that **latent heat** of a pound of wet steam is xL . Similarly, the **total heat** of a pound of wet steam is $h + xL$, and the volume of a pound of wet steam is $xV + \frac{1}{60}(1 - x)$ or approximately equal to xV , because the term $\frac{1}{60}(1 - x)$ is negligibly small except in cases where the steam is so wet as to consist mostly of water. Similarly, the internal energy in a pound of wet steam is

$$I_H = h + x \left\{ L - \frac{P(V - \frac{1}{60})}{778} \right\}. \quad (64)$$

Superheated Steam. When the temperature of steam is higher than that corresponding to saturation as taken from the steam tables, and is, therefore, higher than the standard temperature corresponding to the pressure, the steam is said to be **super-**

heated. In this condition steam begins to depart and differ from its properties in the saturated condition, and when superheated to a very high degree it begins to behave somewhat like a perfect gas.

There are tables of the properties of superheated steam just as there are tables of saturated steam. Tables of superheated steam are very much larger and cover many more pages than those of saturated steam, for the reason that for every pressure there are innumerable values for temperature and also for volume. When dealing with saturated steam there is always only one possible temperature and only one specific volume to be considered. With superheated steam, on the other hand, for a given pressure we may have any temperature **above** that of saturated steam, and corresponding to each temperature there will be, of course, definite values for specific volume and total heat. Like a perfect gas the specific volume or the cubic feet per pound increases with the increase in temperature.

Total heat of superheated steam is obviously greater than the total heat of saturated steam which is not wet by the amount of heat that must be added to dry* saturated steam to produce the required degree of superheat. Thus, since the total heat of dry saturated steam is, as before, $h + L$, the total heat of superheated steam with D degrees of superheat is

$$H_s = h + L + C_p \times D;$$

or if we call the temperature of the superheated steam t_{sup} and t_{sat} is the temperature of saturated steam corresponding to the pressure, we can write, similarly,

$$H_s = h + L + C_p (t_{\text{sup}} - t_{\text{sat}}).$$

The formulas above for superheated steam are for the total heat of steam at constant pressure as shown by the use of C_p . In practically all engineering calculations it is only the condition of total heat at constant pressure that interests us.

* In practice we speak of steam as being dry saturated when it is exactly saturated and has no moisture. It is the condition known simply as saturated steam as regards the properties given in the ordinary steam tables. The dry saturated condition is the boundary between wet steam and superheated steam.

It should be carefully observed that the total heat of superheated steam is the amount of heat required to produce a pound of steam with the required degrees of superheat from water at 32° F.

To obtain the amount of internal energy of a pound of steam (superheated) corresponding to this total heat as stated above, the external work expended in the steaming process must be subtracted; that is,

$$\begin{aligned} I_H &= h + L - \frac{P(V_{\text{sat}} - \frac{1}{60})}{778} + C_p(t_{\text{sup}} - t_{\text{sat}}) - \frac{P(V_{\text{sup}} - V_{\text{sat}})}{778} \\ &= h + L + C_p(t_{\text{sup}} - t_{\text{sat}}) - \frac{P(V_{\text{sup}} - \frac{1}{60})}{778} = H_s - \frac{P(V_{\text{sup}} - \frac{1}{60})}{778}. \end{aligned}$$

The term $\frac{1}{60}$ can be neglected in the equations above for practically all engineering calculations as the maximum error from this is not likely to be more than one in one thousand or $\frac{1}{10}$ per cent.* The accuracy of our steam tables for values of latent and total heat is not established to any greater accuracy. Making this approximation, the equation above becomes

$$I_H = h + L + C_p(t_{\text{sup}} - t_{\text{sat}}) - \left(\frac{P \times V_{\text{sup}}}{778} \right) = H_s - \frac{P \times V_{\text{sup}}}{778}.$$

To make these matters clearer, examine, for example, the properties of superheated steam as given in Marks and Davis' *Steam Tables* † for superheated steam at 165 pounds per square inch absolute pressure and 150° F. superheat. We read as follows:

Press. lbs. per sq. in. abs.	Deg. of Sup. F.
	150
165	<i>t</i>516.0
	<i>V</i>3.43
	<i>H_s</i> or <i>h</i> ‡.....1277.6

* The error in the value of total internal energy due to neglecting the term $\frac{1}{60}$ in the exercise worked out on page 67 is 0.85 B.t.u. per pound (one per cent of the external work), or an error of 0.76 per cent in the final result.

† Page 48, line 13 (First Edition).

‡ In their tables Marks and Davis represent the total heat of superheated steam by *h*. In this book the symbol *H_s* is used for greater clearness and to be consistent with the symbols used in the preceding formulas. They use also *v* for specific volume in place of *V* as above.

The values of h and L for saturated steam at this pressure are respectively 338.2 and 856.8. From the curves given, Fig. 12,* we find that the specific heat of superheated steam at constant pressure (C_p) is 0.552. From these data we could obtain, then, $H_s = h + L + C_p \times 150 = 338.2 + 856.8 + 0.552 \times 150 = 1277.7$

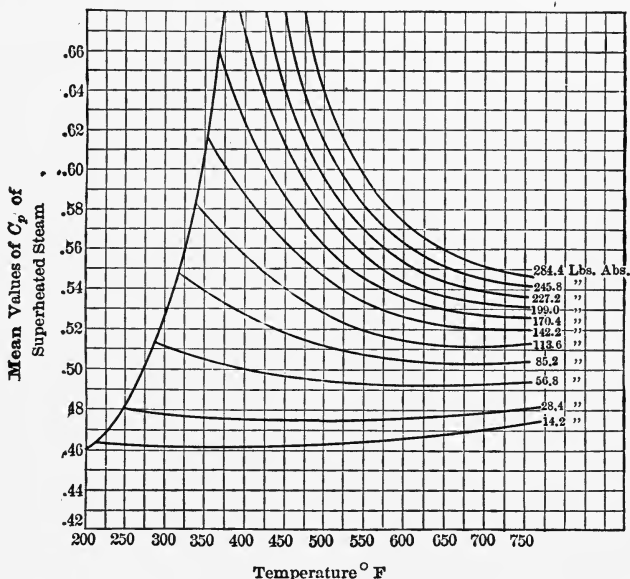


FIG. 12.—Mean Values of C_p Calculated by Integration from Knoblauch and Jakob's Data.

(nearly). Observe that this value agrees with that given in the tables as indicated above. The specific volume (V) is calculated from the empirical formula derived from experimental results and is expressed as follows:

$$V = \left[0.5962 T - p (1 + 0.0014 p) \left(\frac{150,300,000}{T^3} - 0.0833 \right) \right] \frac{1}{p},$$

* The curves for values of C_p , as given in Fig. 12, are for average and not for instantaneous values such as are given in Fig. 13 and also in Fig. 5, page 97, of Marks and Davis' *Tables and Diagrams*. Great caution must be observed in the use of curves of this kind. Those giving instantaneous values can only be used for the value of C_p in the formulas given for the total heat of superheated steam after the average value has been found by integrating the curve representing these values for a given pressure.

where p is in pounds per square inch, V is in cubic feet per pound and $T = t + 460$ is the absolute temperature on the Fahrenheit scale.*

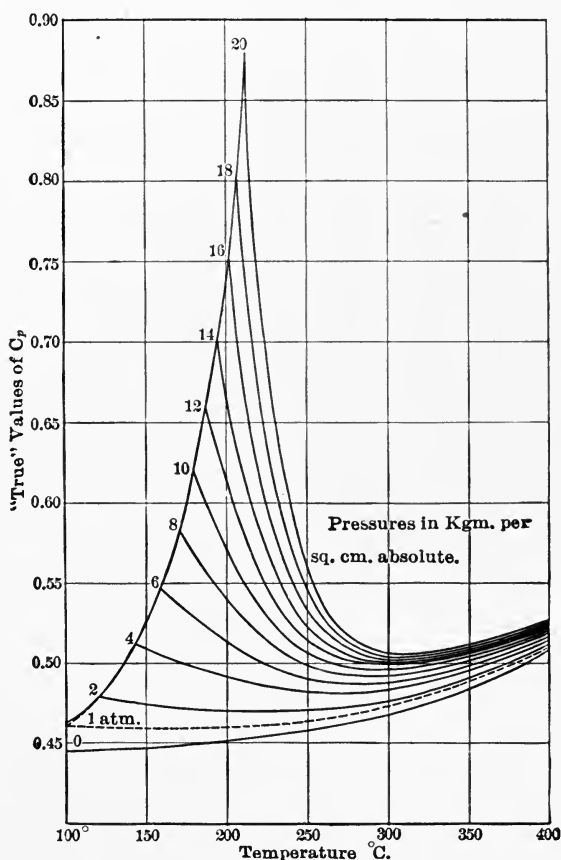


FIG. 13. — Values of the "True" Specific Heat of Superheated Steam.

Drying of Steam by Throttling or Wire-drawing. When steam expands by passing through a very small opening, as, for example, through a valve only partly open in a steam line, the pressure

* The value of γ or $\frac{C_p}{C_v}$ of **superheated** steam used in ordinary engineering calculations is 1.3.

is considerably reduced. Steam engineers usually call this effect **throttling or wire-drawing**. The result of expansion of this kind when the pressure is reduced and no work is done is that if the steam is initially wet it will be drier and if it is initially dry or superheated the degree of superheat will be increased. The reason for this is that the total heat required to form a pound of dry saturated steam (H) is considerably less at low pressure than at high, but obviously the total **quantity** of heat in a pound of steam must be the same after wire-drawing as it was before, neglecting radiation. Now if steam is initially wet and the quality is represented by x_1 , then the total heat in the steam is represented by h_1 plus x_1L_1 , in which h_1 and L_1 represent the heat of the liquid and the latent heat of the steam at the initial pressure. If, also, the quality, heat of liquid and the latent heat of the steam after wire-drawing are represented respectively by x_2 , h_2 and L_2 , then $h_1 + x_1L_1 = h_2 + x_2L_2$. It happens in many engineering calculations that all of the terms in the last equation are known except x_2 and this can be solved as shown in the following equation:

$$x_2 = \frac{x_1L_1 + h_1 - h_2}{L_2}.$$

This drying action of steam in passing through a small opening or an orifice is very well illustrated by steam discharging from a small leak in a high pressure boiler into the atmosphere. It will be observed that no moisture is visible in the steam a few inches from the leak but farther off it becomes condensed by loss of heat, becomes clouded and plainly visible. An important application is also to be found in the throttling calorimeter (page 75).

Determination of the Moisture in Steam. Unless the steam used in the power plant is superheated it is said to be either dry or wet, depending on whether or not it contains water in suspension. The general types of steam calorimeters used to determine the amount of moisture in the steam may be classified under three heads:

1. Throttling or superheating calorimeters.
2. Separating calorimeters.
3. Condensing calorimeters.

Throttling or Superheating Calorimeters. The type of steam calorimeter used most in engineering practice operates by passing a sample of the steam through a very small orifice, in which it is superheated by throttling. A very satisfactory calorimeter of this kind can be made of pipe fittings as illustrated in Fig. 14. It consists of an orifice **O** discharging into a chamber **C**, into which a thermometer **T** is inserted, and a mercury manometer is usually attached to the cock **V₃** for observing the pressure in the calorimeter.

It is most important that all parts of calorimeters of this type, as well as the connections leading to the main steam pipe, should be very thoroughly lagged by a covering of good insulating material. One of the best materials for this use is hair felt, and it is particularly well suited for covering the more or less temporary pipe fittings, valves and nipples through which steam is brought to the calorimeter. Very many throttling calorimeters have been declared useless by engineers and put into the scrap heap merely because the small pipes leading to the calorimeters were not properly lagged, so that there was too much radiation, producing, of course, condensation, so that the calorimeter did not get a true sample. It is obvious that if the entering steam contains too much moisture the drying action due to the throttling in the orifice may not be sufficient to superheat. It may be stated in general that unless there is about 5° to 10° F. of superheat in the calorimeter, or, in other words, unless the temperature on the low pressure side of the orifice is at least about 5° to 10° F. higher than that corresponding to the pressure in the calorimeter, there may be some doubt as to the accuracy of results.* The working limits of throttling calo-

* The same general statement may be made as regards determinations of superheat in engine and turbine tests. Experience has shown that tests made with from 0 to 10 degrees Fahrenheit superheat are not reliable, and that the steam consumption in many cases is not consistent when compared with results

rimeters vary with the initial pressure of the steam. For 35 pounds per square inch absolute pressure the calorimeter ceases to superheat when the percentage of moisture exceeds about 2 per cent; for 150 pounds absolute pressure when the moisture exceeds about 5 per cent; and for 250 pounds absolute pressure when it is in excess of about 7 per cent. For any given pressure the exact limit varies slightly, however, with the pressure in the calorimeter.

In connection with a report on the standardizing of engine tests, the American Society of Mechanical Engineers* published the following instructions regarding the method to be used for obtaining a fair sample of the steam from the main pipes. It is recommended in this report that the calorimeter shall be connected with as short intermediate piping as possible with a so-called **calorimeter nipple** made of $\frac{1}{2}$ -inch pipe and long enough to extend into the steam pipe to within $\frac{1}{2}$ inch of the opposite wall. The end of this nipple is to be plugged so that the steam must enter through not less than twenty $\frac{1}{8}$ -inch holes drilled around and along its length. None of these holes shall be less than $\frac{1}{2}$ inch from the inner side of the steam pipe. The sample of steam should always be taken from a **vertical** pipe as near as possible to the engine, turbine or boiler being tested. A good example of a calorimeter nipple is illustrated in **Fig. 15**.

Never close and do not usually attempt to adjust the discharge valve V_2 without first closing the gage cock V_3 . Unless this precaution is taken, the pressure may be suddenly increased in chamber C, so that if a manometer is used the mercury will be blown out of it; and if, on the other hand, a low-pressure steam gage is used it may be ruined by exposing it to a pressure much beyond its scale.

Usually it is a safe rule to begin to take observations of temperature in calorimeters after the thermometer has indicated a obtained with wet or more highly superheated steam. The errors mentioned when they occur are probably due to the fact that in steam, indicating less than 10 degrees Fahrenheit superheat, water in the liquid state may be taken up in "slugs" and carried along without being entirely evaporated.

* *Proceedings American Society of Mechanical Engineers*, vol. XXI.

maximum value and has again slightly receded from it. The quality or relative dryness of wet steam is easily calculated by the following method and symbols:

p_1 = steam pressure in main, pounds per square inch absolute,

p_2 = steam pressure in calorimeter, pounds per square inch absolute,

t_c = temperature in calorimeter, degrees Fahrenheit,

L_1 and h_1 = heat of vaporization and heat of liquid corresponding to pressure p_1 , B.t.u.,

H_2 and t_2 = total heat (B.t.u.) and temperature (degrees Fahrenheit) corresponding to pressure p_2 ,

C_p = specific heat of superheated steam. Assume 0.47 for low pressures existing in calorimeters,

x_1 = initial quality of steam.

Total heat in a pound of wet steam flowing into orifice is

$$x_1 L_1 + h_1,$$

and after expansion, assuming all the moisture is evaporated, the total heat of the same weight of steam is

$$H_2 + C_p (t_c - t_2).$$

Then assuming no heat losses and putting for C_p its value 0.47 we have

$$\begin{aligned} x_1 L_1 + h_1 &= H_2 + 0.47 (t_c - t_2) \\ x_1 &= \frac{H_2 + 0.47 (t_c - t_2) - h_1}{L_1}. \end{aligned} \quad (65)$$

The following example shows the calculations for finding the quality of steam from the observations taken with a throttling calorimeter:

Example. Steam at a pressure of 100 pounds per square inch absolute passes through a throttling calorimeter. In the calorimeter the temperature of the steam becomes 243° F. and the pressure 15 pounds per square inch absolute. Find the quality.

Solution. By taking values directly from tables of properties of superheated steam,* the total heat of the steam in the calorimeter at 15 pounds per square inch absolute pressure and 243° F. is 1164.8 B.t.u. per pound or can be calculated as follows:

$$\begin{aligned} H + C_p(t_c - t_2) &= 1150.7 + 0.47(243 - 213) \\ &= 1164.8 \text{ B.t.u. per pound.} \end{aligned}$$

(Note. t_c = temperature in calorimeter and t_2 = temperature corresponding to calorimeter pressure.)

The total heat of the steam before entering the calorimeter is $h + xL$. At 100 pounds per square inch absolute pressure, this is $298.3 + 888.0x$ in B.t.u. per pound. Since the heat in the steam per pound in the calorimeter is obviously the same as before it entered the instrument, we can equate as follows:

$$\begin{aligned} 298.3 + 888.0x &= 1164.8 \\ x &= 0.976 \end{aligned}$$

or the steam is 2.4 per cent wet.

Barrus Throttling Calorimeter. This is an important variation from the type of throttling calorimeter shown in Fig. 14 and has been quite widely introduced by Mr. George H. Barrus. In this apparatus the temperature of the steam admitted to the calorimeter is observed instead of the pressure and a very free exhaust is provided so that the pressure in the calorimeter is atmospheric. This arrangement simplifies very much the observations to be taken, as the quality of the steam x_1 can be calculated by equation (65) by observing only the two temperatures t_1 and t_c , taken respectively on the high and low pressure sides of the orifice in the calorimeter. This calorimeter is illustrated in Fig. 15. The two thermometers required are shown in the figure. Arrows indicate the path of the steam.†

The orifice in such calorimeters is usually made about $\frac{3}{8}$ inch in diameter, and for this size of orifice the weight of steam ‡

* Marks and Davis' *Steam Tables and Diagrams* (1st ed.), page 24.

† *Transactions of American Society of Mechanical Engineers*, vol. XI, page 790.

‡ In boiler tests corrections should be made for the steam discharged from the steam calorimeters.

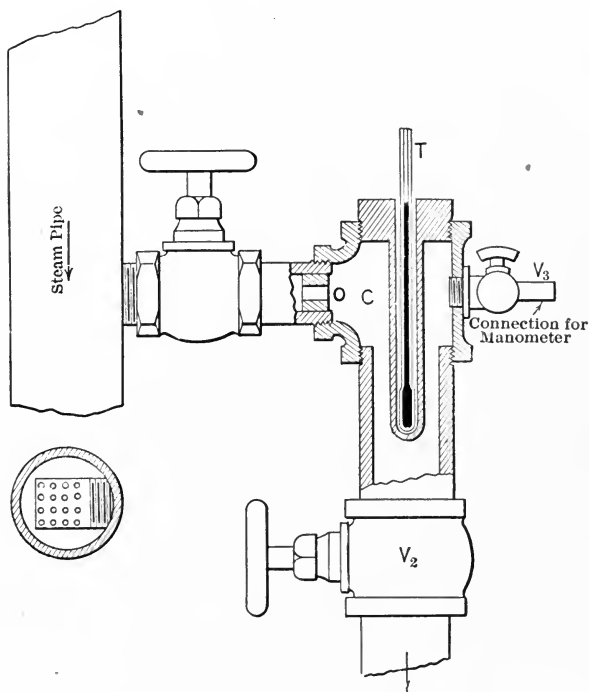


FIG. 14. — Simple Throttling Steam Calorimeter.

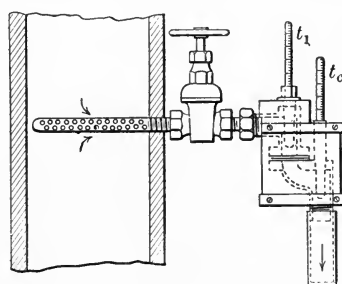


FIG. 15. — Barrus' Throttling Steam Calorimeter.

discharged per hour at 175 pounds per square inch absolute pressure is about 60 pounds. It is important that the orifice should always be kept clean, because if it becomes obstructed there will be a reduced quantity of steam passing through the

instrument, making the error due to radiation relatively more important.

In order to free the orifice from dirt or other obstructions the connecting pipe to be used for attaching the calorimeter to the main steam pipe should be blown out thoroughly with steam before the calorimeter is put in place. The connecting pipe and valve should be covered with hair felting not less than $\frac{3}{4}$ inch thick. It is desirable also that there should be no leak at any point about the apparatus, either in the stuffing-box of the supply valve, the pipe joints or the union.

With the help of a diagram* giving the quality of steam directly the Barrus calorimeter is particularly well suited for use in power plants, where the quality of the steam is entered regularly on the log sheets. The percentage of moisture is obtained immediately from two observations without any calculations.

Separating Calorimeters. It was explained on page 76 that throttling calorimeters cannot be used for the determination of the quality of steam when for comparatively low pressures the moisture is in excess of 2 per cent, and when for average boiler pressures in modern engineering practice it exceeds 5 per cent. For higher percentages of moisture than these low limits separating calorimeters are most generally used. In these instruments the water is removed from the sample of steam by **mechanical separation** just as it is done in the ordinary steam separator installed in the steam mains of a power plant. There is provided, of course, a device for determining, while the calorimeter is in operation, usually by means of a calibrating gage glass, the amount of moisture collected. This mechanical separation depends for its action on changing very abruptly the direction of flow and reducing the velocity of the wet steam. Then, since the moisture (water) is nearly 300 times as heavy as steam at the usual pressures delivered to the engine, the moisture will be deposited because of its greater inertia.

When a U-tube manometer is used to determine the pressure in a calorimeter of the type illustrated in **Fig. 14**, this pressure

* See page 117, and also Moyer's *Power Plant Testing*, 2d edition, pages 58-60.

can be obtained very accurately, and an excellent means is provided for **calibrating the thermometer** in the calorimeter just as it is to be used. The calibration would be made, of course, by the method of comparing with the temperature corresponding to known pressures. In order to avoid having superheated steam in the calorimeter for this calibration the felt or similar material usually needed for covering the valves and nipples between the main steam pipe and the calorimeter should be kept saturated with cold water.

Fig. 16 illustrates a form of separating calorimeter having a steam jacketing space which receives live steam at the same temperature as the sample. Steam is supplied through a pipe **A**, discharging into a cup **B**. Here the direction of the flow is changed through nearly 180 degrees, causing the moisture to be thrown outward through the meshes in the cup into the vessel **V**. The dry steam passes upward through the spaces between the webs **W**, into the top of the outside jacketing chamber **J**, and is finally discharged from the bottom of this steam jacket through

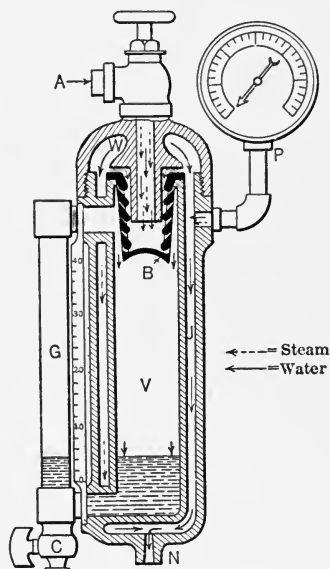


FIG. 16.—Separating Calorimeter.

the nozzle **N**. This nozzle is considerably smaller than any other section through which the steam flows, so that there is no appreciable difference between the pressures in the calorimeter proper and the jacket. The scale opposite the gage glass **G** is graduated to show in hundredths of a pound at the temperature corresponding to steam at ordinary working pressures, the variation of the level of the water accumulating. A steam pressure gage **P** indicates the pressure in the jacket **J**, and since the flow of steam through the nozzle **N** is roughly

proportional to the pressure, another scale in addition to the one reading pressures is provided at the outer edge of the dial. A petcock **C** is used for draining the water from the instrument, and by weighing the water collected corresponding to a given difference in the level in the gage **G** the graduated scale can be readily calibrated. Too much reliance should not be placed on the readings for the flow of steam as indicated by the gage **P** unless it is frequently calibrated. Usually it is very little trouble to connect a tube to the nozzle **N** and condense the steam discharged in a large pail nearly filled with water. When a test for quality is to be made by this method the pail nearly filled with cold water is carefully weighed, and then at the moment when the level of the water in the water gage **G** has been observed the tube attached to the nozzle **N** is immediately placed under the surface of the water in the pail. The test should be stopped before the water gets so hot that some weight is lost by "steaming." The gage **P** is generally calibrated to read pounds of steam flowing in ten minutes. For the best accuracy it is desirable to use a pail with a tightly fitting cover into which a hole just the size of the tube has been cut.

If W is the weight of dry steam flowing through the orifice **N** and w is the weight of moisture separated, the **quality** of the steam is

$$x = \frac{W}{W + w}. \quad (66)$$

Condensing or Barrel Calorimeter. For steam having a large percentage of moisture (over 5 per cent) the condensing or barrel calorimeter will give fairly good results if properly used. In its simplest form it consists of a barrel placed on a platform scale and containing a known weight of cold water. The steam is introduced by a pipe reaching nearly to the bottom of the barrel. The condensation of the steam raises the temperature of the water, the loss of heat by the wet steam being equal to the gain of heat by the cold water. It will, therefore, be necessary to observe the initial and final weights, the initial and

final temperatures of the water in the barrel and the temperature of the steam.

Let W = original weight of cold water, pounds.

w = weight of wet steam introduced, pounds.

t_1 = temperature of cold water, degrees Fahrenheit.

t_2 = temperature of water after introducing steam.

t_s = temperature of steam.

L = latent heat of steam at temperature t_s .

x = quality of steam.

Then,

Heat lost by wet steam = heat gained by water.

$$wxL + w(t_s - t_2) = W(t_2 - t_1)$$

$$x = \frac{W(t_2 - t_1) - w(t_s - t_2)}{wL}. \quad (67)$$

The accuracy of the results depends obviously upon the accuracy of the observation, upon thorough stirring of the water so that a uniform temperature is obtained and upon the length of time required. The time should be just long enough to obtain accurate differences in weights and temperatures; otherwise, losses by radiation will make the results much too low.

Equivalent Evaporation and Factor of Evaporation. For the comparison of the total amounts of heat used for generating steam (saturated or superheated) under unlike conditions it is necessary to take into account the temperature t_0 at which the water is put into the boiler as well as also the pressure P at which the steam is formed.* These data are of much importance in comparing the results of steam boiler tests. The basis of this comparison is the condition of water initially at the boiling point for "atmospheric" pressure or at 14.7 pounds per square inch; that is, at 212° F. and with steaming taking place at the same temperature. For this standard condition, then,

* As the pressure P increases the total heat of the steam also increases; but as the initial temperature of the water ("feed temperature") increases the value of the heat of the liquid decreases.

$h = 0$ and $H = L = 970.4$ B.t.u. per pound. Evaporation under these conditions is described as,

“**from** (a feed-water temperature of) **and at** (a pressure corresponding to the temperature of) **212° F.**”

To illustrate the application of a comparison with this standard condition let it be required to compare it with the amount of heat required to generate steam at a pressure of 200 pounds per square inch absolute with the temperature of the water supplied (feed water) at 190° F.

For $P = 200$ pounds per square inch absolute the heat of the liquid $h = 354.9$ B.t.u. per pound and the heat of evaporation (L) is 843.2 B.t.u. per pound. For $t = 190^\circ$ F. the heat of the liquid (h_0) is 157.9 B.t.u. per pound. The total heat actually required in generating steam at these conditions is, therefore,

$$843.2 + (354.9 - 157.9) = 1040.2 \text{ B.t.u. per pound.}$$

The ratio of the total heat actually used for evaporation to that necessary for the condition defined by “from and at 212° F.” is called the **factor of evaporation**. In this case it is the value

$$1040.2 \div 970.4 = 1.07.$$

If we write F for factor of evaporation, h and L respectively the heats of the liquid and of evaporation corresponding to the steam pressure and h_0 the heat of the liquid corresponding to the temperature of feed water, then

$$F = \frac{L + (h - h_0)}{970.4}.$$

The actual evaporation of a boiler (expressed usually in pounds of steam per hour) multiplied by the factor of evaporation is called the **equivalent evaporation**.

PROBLEMS

1. Dry and saturated steam has a pressure of 100 lbs. per sq. in. absolute. What is the temperature of the steam? *Ans.* 327.8° F.
2. What is the volume per pound of this steam? *Ans.* 4.429 cu. ft.
3. What is the heat of the liquid per pound of this steam? *Ans.* 298.3 B.t.u.

4. What is the latent heat per pound of this steam?

Ans. 888.0 B.t.u.

5. What is the total heat above 32° F. per pound of this steam?

Ans. 1186.3 B.t.u.

6. Dry and saturated steam has a temperature of 300° F. What is its pressure?

Ans. 67.0 lbs. per sq. in. absolute.

7. How many British thermal units would be required to raise the temperature of 1 lb. of water from 32 degrees to the boiling point as stated in problem 6?

Ans. 269.6 B.t.u.

8. How many British thermal units are required to evaporate 1 lb. of this water into dry and saturated steam under the conditions of problem 6?

Ans. 909.5 B.t.u.

9. How many British thermal units are required to generate 1 lb. of dry and saturated steam from water at 32° F. under the conditions of problem 6?

Ans. 1179.1 B.t.u.

10. A closed tank contains 9 cu. ft. of dry and saturated steam at a pressure of 150 lbs. per sq. in. absolute.

- (a) What is its temperature?

Ans. 358.5° F.

- (b) How many pounds of steam does the tank contain?

Ans. 2.988 lbs.

11. A boiler generates dry and saturated steam under a pressure of 200 lbs. per sq. in. absolute. The feed water enters the boiler at 60° F.

- (a) What is the temperature of the steam?

Ans. 381.9° F.

- (b) How many British thermal units are required to generate 1 lb. of this steam if this feed water is admitted at 32° F.?

Ans. 1198.1 B.t.u.

- (c) How many British thermal units are required to raise the temperature of 1 lb. of water from 32° to 60° F.?

Ans. 28.08 B.t.u.

- (d) How many British thermal units are required to generate 1 lb. of this steam from feed water at 60 degrees into the steam at the pressure stated at the beginning of this problem?

Ans. 1170.02 B.t.u.

12. One pound of dry and saturated steam is under a pressure of 250 lbs. per sq. in. absolute.

- (a) What is its internal energy of evaporation?

Ans. 742.0 B.t.u.

- (b) What is its total internal energy above 32° F.?

Ans. 1116.4 B.t.u.

- (c) How much external work was done during its formation from 32° F.?

Ans. 85.1 B.t.u.

- (d) How much external work was done during the evaporation?

Ans. 84.3 B.t.u.

- (e) How much external work was done during the change in temperature of the water from 32 degrees to the boiling point corresponding to the pressure? *Ans.* 0.8 B.t.u.

13. Dry and saturated steam is generated in a boiler and has a temperature of 400° F. The feed water enters the boiler at 200° F.

- (a) What pressure is **carried** in the boiler?

Ans. 247.1 lbs. per sq. in. absolute.

- (b) What is the total heat supplied to generate 1 lb. of this steam?

Ans. 1033.36 B.t.u.

- (c) How much external work was done during its formation?

Ans. 84.97 B.t.u.

- (d) How much heat was used in increasing the internal energy?

Ans. 948.39 B.t.u.

Check this by $(h_2 - h_1 + I_L)$ noting that this assumes no external work done in the heating of the liquid.

Ans. 949.16 B.t.u.

14. One pound of steam at a pressure of 100 lbs. per sq. in. absolute has a quality of 90 per cent dry. What is its temperature? *Ans.* 327.8° F.

15. How many British thermal units would be required to raise the pound of steam in the above problem from 32° F. to the boiling point corresponding to the pressure stated? *Ans.* 298.3 B.t.u.

16. What would be the volume of a pound of steam for the conditions stated in problem 14? *Ans.* 3.99 cu. ft.

17. How many heat units (latent heat) are required to evaporate the steam in problem 14? *Ans.* 799.2 B.t.u.

18. What is the amount of the total heat (above 32° F.) of the steam in problem 14? *Ans.* 1097.5 B.t.u.

19. What would be the external work of evaporation of the steam in problem 14? *Ans.* 73.26 B.t.u.

20. How much external work (above 32° F.) is done in making steam as in problem 14? *Ans.* 73.56 B.t.u.

21. What is the internal energy of evaporation of the steam in problem 14? *Ans.* 725.9 B.t.u.

22. What is the total internal energy of the steam in problem 14?

Ans. 1023.94 B.t.u.

23. A tank contains 9 cu. ft. of steam at 100 lbs. per sq. in. absolute pressure which has a quality of 95 per cent. How many pounds of steam does the tank contain? *Ans.* 2.14 lbs.

24. Two pounds of steam have a volume of 8 cu. ft. at a pressure of 100 lbs. per sq. in. absolute. What is the quality? *Ans.* 90.5 per cent dry.

Calculate its total heat above 32° F. *Ans.* 2003.8 B.t.u.

25. One pound of steam having a quality of 0.95 has a temperature of 325° F. What is the pressure? *Ans.* 96.15 lbs. per sq. in. absolute.

26. One pound of steam at a pressure 225 lbs. per sq. in. absolute has a temperature of 441.9°F . Is it superheated or saturated? How many degrees of superheat has it? *Ans.* 50°F . sup.

What is the total heat required to generate such steam from water at 32°F .? *Ans.* 1232.7 B.t.u.

What is its volume? *Ans.* 2.23 cu. ft.

How much external work was done (above 32°F .) in generating it? *Ans.* 92.2 B.t.u.

How much internal energy above 32°F . does it contain? *Ans.* 1140.5 B.t.u.

27. One pound of steam at a pressure of 300 lbs. per sq. in. absolute has a volume of 1.80 cu. ft.

Is it saturated or superheated?

What is its temperature? *Ans.* 507.5°F .

How much superheat has it? *Ans.* 90°F .

How much is its total heat above 32°F .? *Ans.* 1262.8 B.t.u.

What is its total internal energy? *Ans.* 1163.8 B.t.u.

28. Steam in a steam pipe has pressure of 110.3 lbs. per sq. in. by the gage. A thermometer in the steam registers 385°F . Atmospheric pressure is 14.7 lbs. per sq. in. absolute. Is the steam superheated, and if superheated how many degrees?

29. Steam at a pressure of 200 lbs. per sq. in. absolute passes through a throttling calorimeter. After expansion into the calorimeter the temperature of this steam is 250°F . and the pressure 15 lbs. per sq. in. absolute. What is the quality? *Ans.* 0.965.

30. Steam at a temperature of 325°F . passes through a throttling calorimeter. In the calorimeter the steam has a pressure of 16 lbs. per sq. in. absolute and a temperature of 236.3°F . What is the quality? *Ans.* 0.973.

31. Steam at 150 lbs. per sq. in. absolute pressure passes through a throttling calorimeter. Assuming that the lowest conditions in the calorimeter for measuring the quality is 10°F . superheat and the pressure in the calorimeter is 15 lbs. per sq. in. absolute, what is the largest percentage of wetness the calorimeter is capable of measuring under the above conditions? *Ans.* 4.3 per cent wet or a quality of 0.957.

32. In a ten-minute test of a separating calorimeter the quantity of dry steam passing through the orifice is 9 lbs. The quantity of water separated was 1 lb. What was the quality? *Ans.* 0.90.

33. A barrel contains 400 lbs. of water at a temperature of 50°F . Into this water steam at a pressure of 125 lbs. per sq. in. absolute is admitted until the temperature of the water and condensed steam in the barrel reaches a temperature of 100°F . The weight of the water in the barrel was then 418.5 lbs. What was the quality? *Ans.* 0.958.

CHAPTER VI

PRACTICAL APPLICATIONS OF THERMODYNAMICS TO THERMAL MACHINERY

Refrigerating Machines or Heat Pumps. By a refrigerating machine or heat pump is meant a machine which will carry heat from a cold to a hotter body.* This, as the second law of thermodynamics asserts, cannot be done by a self-acting process, but it can be done by the expenditure of mechanical work. Any heat engine will serve as a heat pump if it be forced to trace its indicator diagram backward, so that the area of the

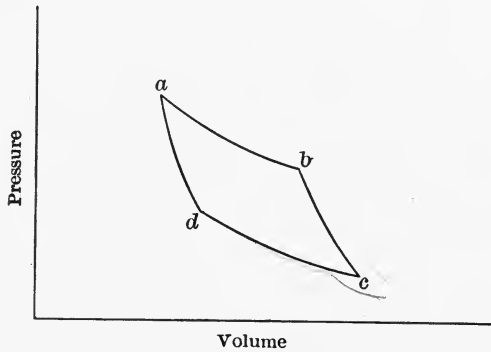


FIG. 17. — Pressure-volume Diagram of Carnot Cycle.

diagram represents work **spent on**, instead of **done by**, the working substance. Heat is then **taken in from the cold body** and heat is rejected to the hot body.

Take, for instance, the Carnot cycle, using air as working substance (Fig. 17), and let the cycle be performed in the order **dcba**, so that the area of the diagram is negative, and represents work spent upon the machine. In the stage **dc**, which is

* This statement is not at variance with our knowledge that heat does not *flow* of itself from a cold body to a hotter body.

isothermal expansion in contact with the cold body **R** (as in Fig. 9, page 45), the air takes in a quantity of heat from **R** equal to $MRT_2 \log_e r$ (equation (29)), and in stage **ba** it gives out to the hot body **H** a quantity of heat equal to $MRT_1 \log_e r$. There is no transfer of heat in stages **cb** and **ad**. Thus **R**, the cold body, is constantly being drawn upon for heat and can therefore be maintained at a temperature lower than its surroundings. In an actual refrigerating machine **operating with air**, of the kind that might be used for making ice, the cold body **R** consists of a coil of pipe through which brine circulates while "working" air is brought into contact with the outside of the pipe. The brine is kept, by the action of the machine, at a temperature below 32°F . and is used in its turn to extract heat by conduction from the water which is to be frozen to make ice. The "cooler" **H**, which is the relatively hot body, is kept at as low a temperature as possible by means of circulating water, which absorbs the heat rejected to **H** by the "working" air.

The use of a regenerator, as in Stirling's engine (page 55), may be resorted to in place of the two adiabatic stages in the Carnot cycle just explained with the advantage of making the machine much less bulky. Refrigerating machines of this kind using air as working substance, with a regenerator, were introduced by Dr. A. C. Kirk and have been widely used.* The working air is completely enclosed, which allows it to be in a compressed state throughout, so that even its lowest pressure is much above that of the atmosphere. This makes a greater mass of air pass through the cycle in each revolution of the machine, and hence increases the performance of a machine of given size.

In another class of refrigerating machines the working substance, instead of being air, consists of a liquid and its vapor, and the action proceeds by alternate evaporation under a low pressure and condensation under a relatively high pressure. A

* See Kirk, *On the Mechanical Production of Cold*, *Proc. Inst. of C. E.*, vol. XXXVII, 1874. Also lectures on Heat and its Mechanical Applications, in the same proceedings for 1884.

liquid must be chosen which evaporates at the lower extreme of temperature under a pressure which is not so low as to make the bulk of the engine excessive. Ammonia, ether, sulphurous acid, and other volatile liquids have been used. Ether machines are inconveniently bulky and cannot be used to produce intense cold, for the pressure of that vapor is only about 1.3 pounds per square inch at 4° F., and to make it evaporate at any temperature nearly as low as this would require the cylinder to be excessively large in proportion to the performance. This would not only make the machine clumsy and costly, but would involve much waste of power in mechanical friction. The tendency of the air outside to leak into the machine is another practical objection to the use of so low a pressure. With ammonia a distinctly lower limit of temperature is practicable: the pressures are rather high and the apparatus is compact.

The standard systems of mechanical refrigeration are: *

(A) The dense-air system, so-called because the air which is the medium is never allowed to fall to atmospheric pressure, so as to reduce the size of the cylinders and pipes through which a given weight is circulating.

(B) The compression system, using ammonia, carbon dioxide or sulphur dioxide, and so-called to distinguish it from the third system, because a compressor is used to raise the pressure of the vapor and deliver it to the condenser after removing it from the evaporator.

(C) The absorption system, using ammonia, and so-called because a weak water solution removes vapor from the evaporator by **absorption**, the richer aqua ammonia so formed being pumped into a high-pressure chamber called a generator in communication with the condenser, where the ammonia is discharged from the liquid solution to the condenser by heating the generator, to which the solution is delivered by the pump.

No matter what system is used, circulating water is employed to receive the heat, the temperature of which limits the highest

* Lucke's *Engineering Thermodynamics*, page 1148.

temperature allowable in the system and indirectly the highest pressure.

The **dense or closed air system** is illustrated in Fig. 18, in

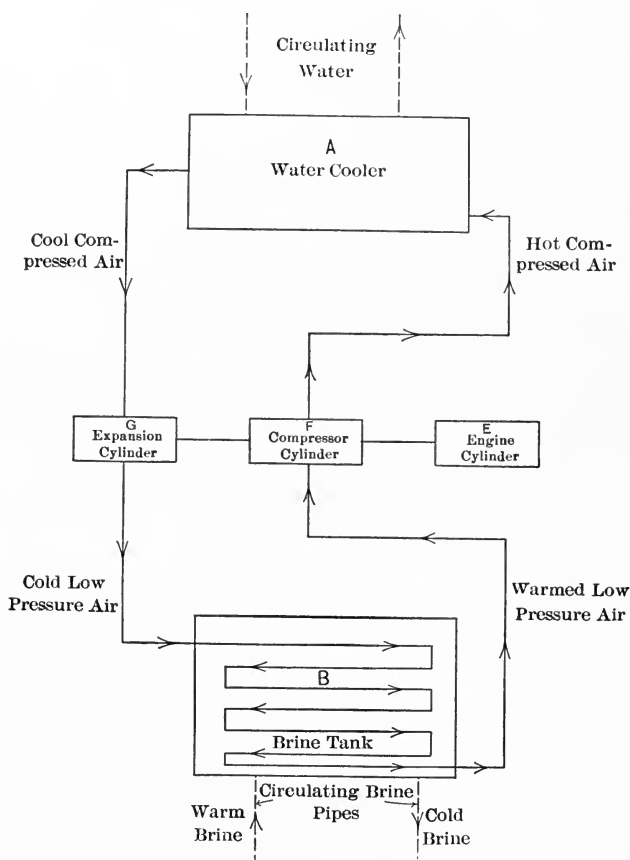


FIG. 18.—Dense Air System of Refrigeration.

which air previously dried of moisture is continuously circulated. The engine cylinder, **E**, furnishes power * to drive the compressor

* Since the work done by the expansion of the cool-compressed air is less than that necessary for the compressing of the air taken from the brine coils through the same pressure conditions, a means must be employed to make up for the difference, and for this purpose the engine cylinder is used.

cylinder, **F**. This cylinder delivers hot-compressed air into a cooler, **A**, where it is cooled, and then passed on to the expansion cylinder, **G** (tandem-connected to both, the compressor, **F**, and to the engine cylinder, **E**), which in turn sends cold low-pressure air first through the refrigerating coils in the brine tank, **B**, and then back to the compressor cylinder, **F**; thus the air cycle is completed. The courses of the circulating water

and also of the brine are shown by the dotted lines.

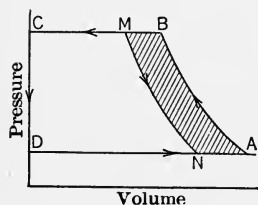


FIG. 19. — Pressure-volume Diagram of Dense Air Cycle of Refrigeration.

The dense-air cycle in a pressure-volume diagram is represented in Fig. 19, in which **BC** is the delivered volume of hot-compressed air; **CM** is the volume of cooled air admitted in the expansion cylinder; **MB** the reduction in volume due to the water cooler; **MN**, the expansion; **NA** the refrigeration or heating of the air by the brine, and **AB** the compression. This operation is but a reproduction of that previously described.

The **compression system** for ammonia or similar condensable vapors is shown in Fig. 20. The figure only illustrates the essential members of a complete compression refrigerating system. **B** represents the direct-expansion coil in which the working medium is evaporated; **F**, the compressor or pump for increasing the pressure of the gasified ammonia; **E**, the engine cylinder, — the source of power; **W**, the condenser for cooling and liquefying the gasified ammonia; and **V** a throttling valve by which the flow of liquefied ammonia under the condenser pressure is controlled as it flows from the receiver **R** to the expansion coils; **B** (the brine tank), in which a materially lower pressure is maintained by the pump or compressor in order that the working medium may boil at a sufficiently low temperature to take heat from and consequently refrigerate the brine which is already cooled.

These descriptions of the refrigeration systems will serve as a foundation for a general understanding of refrigerating.

COEFFICIENT OF PERFORMANCE OF REFRIGERATING MACHINES

$$\frac{\text{Heat extracted from the cold body.}}{\text{Work expended}}$$

This ratio may be taken as a coefficient of performance in estimating the merits of a refrigerating machine from the thermo-

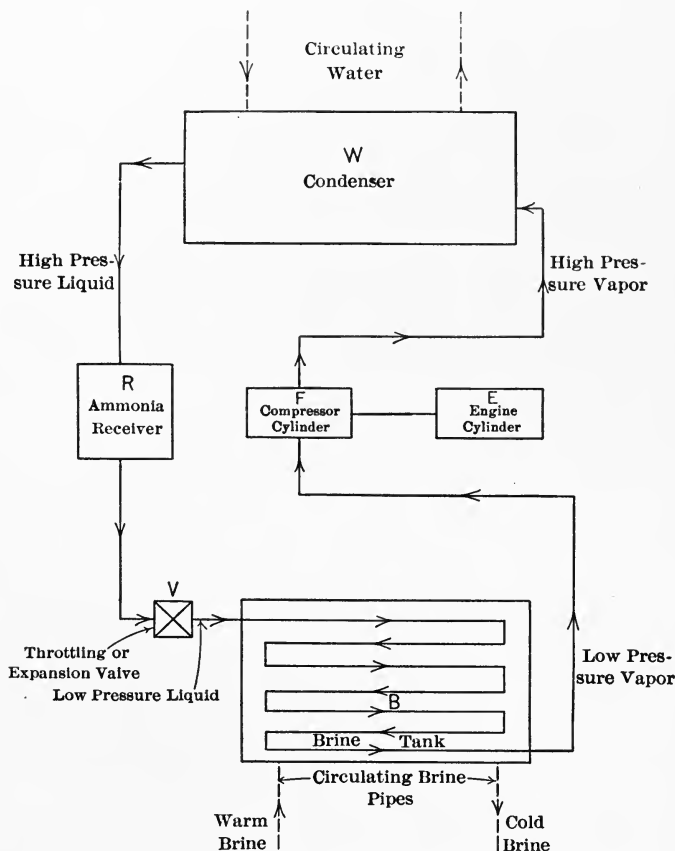


FIG. 20. — Compression System of Refrigeration.

dynamic point of view. When the limits of temperature T_1 and T_2 are assigned it is very easy to show by a slight variation of the argument used in Chapter IV that no refrigerating machine

can have a higher coefficient of performance than one which is reversible according to the Carnot method. For let a refrigerating machine **S** be driven by another **R** which is reversible and is used as a heat-engine in driving **S**. Then if **S** had a higher coefficient of performance than **R** it would take from the cold body more heat than **R** (working reversed) rejects to the cold body, and hence the double machine, although purely self-acting, would go on extracting heat from the cold body in violation of the Second Law (page 3). Reversibility, then, is the test of perfection in a refrigerating machine just as it is in a heat-engine.

When a reversible refrigerating machine takes in all its heat, namely Q_c at T_2 and rejects all, namely Q_a at T_1 and if we represent the heat equivalent of the work done by $W = Q_a - Q_c$, then the coefficient of performance is as already defined,

$$\frac{Q_c}{W} = \frac{Q_c}{Q_a - Q_c} = \frac{T_2}{T_1 - T_2}.$$

Hence — and the inference is highly important in practice — the smaller the range of temperature, the better is the performance. To cool a large mass of any substance through a few degrees will require much less expenditure of energy than to cool one-fifth of the mass through five times as many degrees, although the amount of heat extracted is the same in both cases. If we wish to cool a large quantity, say of water or of air, it is better to do it by the direct action of a refrigerating engine working through the desired range of temperature, than to cool a portion through a wider range and then let this mix with the rest. This is only another instance of a wide, general principle, of which we have had examples before, that any mixture or contact of substances at different temperatures is thermodynamically wasteful because the interchange of heat between them is irreversible. An ice-making machine, for example, should have its lower limit of temperature only so much lower than 32°F. as will allow heat to be conducted to the working fluid with sufficient rapidity from the water that is to be frozen.

COMPRESSED AIR

Air when compressed may be used as the working medium in an engine, in exactly the same way as steam. Furthermore, it is an agent for the transmission of power and can be distributed very easily from a central station for the purpose of driving engines, operating quarry drills and various other pneumatic tools. The type of machine used for the compression of air is that known as a piston-compressor, and consists of a cylinder provided with valves and within which there is a reciprocating piston.

Inasmuch as the work performed in the air-cylinder of a compressor depends on so many variable conditions, it can only be studied successfully from an indicator diagram. Imagine in such a compressor, that the compression is performed very slowly in a conducting cylinder, so that the air within may lose heat by conduction to the atmosphere as fast as heat is generated by compression; the process will in that case be isothermal, at the temperature of the atmosphere. Imagine further that the compressed air is distributed to be used in compressed air motors* or engines without a change of temperature, and that the process of expansion in the compressed air motors or engines is also indefinitely slow and consequently isothermal. In that case (if we neglect the losses caused by friction in pipes) there could be no waste of power in the whole process of transmission. The indicator diagram would then be the same per pound of air in the compressor as in the air motor or engine, although the course of the cycle would be the reverse — that is, it would retrace itself.

Imagine, on the other hand, that compression and expansion are both adiabatic — a state of things which would be approximately true, if expansion and compression were performed very quickly. The diagram of the compression, **Fig. 21**, is **FCBE** and that of the air engine as in **Fig. 22**, is **EADF**, and, therefore,

* Compressed air motors are really engines just like steam engines but use compressed air instead of steam.

CB and **AD** are both adiabatic lines. The change of volume of the compressed air from that of **EB** to **EA** occurs through its cooling in the distributing pipes, from the temperature produced by adiabatic compression down to the temperature of the atmosphere. Suppose both diagrams of compressor and of

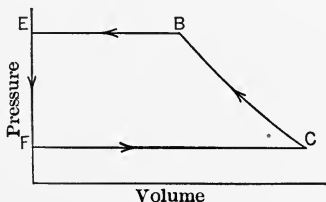


FIG. 21.—Diagram of Compressor.

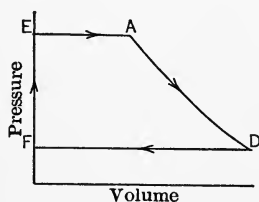


FIG. 22.—Diagram of Air Engine.

air engine be superimposed as in **Fig. 23**, and then sketch an imaginary isothermal line between the points **A** and **C**, both of which are at atmospheric conditions as regards temperature.

This simple sketch shows that the use of adiabatic compression causes a waste of power which is measured by the area

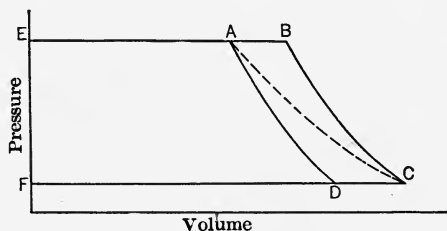


FIG. 23.—Superimposed Diagrams of Figs. 21 and 22.

ABC, while the use of the adiabatic expansion in the air engine involves a further waste, shown by the area **ACD**.

In practice the compression cannot be made strictly isothermal for want of time, — the operation of the piston would be too slow for practice. The difference between isothermal and adiabatic compression (and expansion) can be very clearly shown graphically as in **Figs. 24** and **25**. In this illustration the terminal points are correctly placed for a certain ratio for both compression and expansion. Note that in the compressing diagram

(Fig. 24), the area between the two curves **ABC** represents the work lost in compressing due to heating, and the area between the two curves, **ACMNF** (in Fig. 25), shows the work lost by cooling during the expansion. The isothermal curve **AC** will be the same for both cases. Illustrations of this sort show the

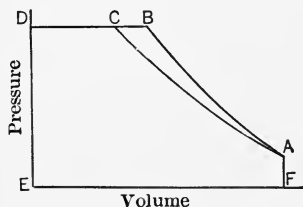


FIG. 24. — Compression Diagram.

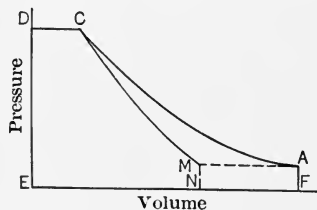


FIG. 25. — Expansion Diagram.

effect of reheating before expansion, cooling before compression, heating during expansion, etc.

The temperature of the air is prevented as far as possible from rising during the compression by injecting water into the compressing cylinder, and in this way both the isothermal and adiabatic curves will change. The curves which would have been $PV = \text{a constant}$, if isothermal and $PV^{1.4} = \text{a constant}$, if adiabatic will be very much modified. In perfectly adiabatic conditions the exponent " n " = 1.40 for air, but in practice the compressor cylinders are water-jacketed, and thereby part of the heat of compression is conducted away, so that " n " becomes less than 1.40. This value of " n " varies with conditions; generally the value is taken as 1.2.

The problem of economy, obviously, becomes one of abstracting the heat generated in the air during the process of compression. As previously mentioned, this is **partially** accomplished by water-jacketing the cylinders, and also by water injection. Nevertheless, owing to the short interval within which the compression takes place, and the comparatively small volume of air actually in contact with the cylinder walls, very little really occurs. The practical impossibility of proper cooling to prevent waste of energy leads to the alternative of discharging air from one cylinder after partial compression has been effected,

into a so-called **inter-cooler**, intended for removing the heat generated during the first compression, and then compressing the air to the final pressure in another cylinder. This operation is termed "**two-stage**" compression and when repeated one or more times for high pressures, the term "**multi-stage**" compression applies.

Referring to **Fig. 26** and assuming the compression in a two-stage compressor to be adiabatic for each cylinder, the compression

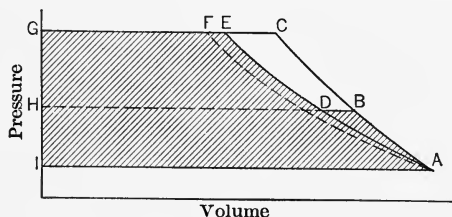


FIG. 26. — Indicator Diagram of Two-stage Air Compressor.

sion curve is represented by the broken line **ABDE**; the compression proceeds adiabatically in the first or low-pressure cylinder to **B**; the air is then taken to a cooler and cooled under practically constant pressure until its initial temperature is almost reached, and its volume reduced from **HB** to **HD**; it is then introduced to the second or high-pressure cylinder and compressed adiabatically along the line **DE** to the final pressure condition that was desired. It is seen that the compression curve approaches the isothermal line **FA**.* The isothermal condition is obviously desired and, in consequence, air-machines are built to approach that condition as nearly as possible.

Numerous devices may be applied by the engineer to make the expansion curves in his air engines approximate more nearly to the isothermal line; that is, he may use a preheater or inject hot water; or use a compound engine, allowing the air time to take up enough heat to restore it more or less nearly to atmospheric temperature between one stage of expansion and the next. By these means the **efficiency of the transmitting system** as a whole (neglecting all losses due to friction in the distributing

* The line **FE** represents further cooling.

pipes, in the valves of the engines, etc.), may be considerably increased.

There is, however, another point to be considered. If the temperature be allowed to fall materially during expansion, the expanding air tends to deposit dew or even snow. To prevent this the practice is often followed of passing the compressed air through a stove or "preheater" in order to raise its temperature just before it is allowed to expand and so prevent the deposit of frozen moisture. When preheaters are used the extra heat which they supply is, of course, itself partly converted into work.*

PROBLEMS

1. If 200 cu. ft. of free air per minute (sea-level) is compressed isothermally and then delivered into a receiver, the internal pressure of which is 102.9 lbs. per sq. in. absolute, find the theoretical horse-power required.

Ans. 24.93 h.p.

2. What will be the net work in foot-pounds per stroke by an air compressor displacing 3 cu. ft. per stroke, compressing air from an atmospheric pressure of 15 lbs. per sq. in. absolute, to a gage pressure of 75 lbs.? (Isothermal.)

Ans. 11,595.

3. What horse-power will be needed to compress adiabatically 1500 cu. ft. of free air per minute to a gage pressure of 58.8 lbs., when n equals 1.4?

Ans. 197 h.p.

4. A compressed-air motor without clearance takes air at a condition of 200 lbs. per sq. in. (gage) and operates under a cut-off at one-fourth stroke. What is the work in foot-pounds that can be obtained per cubic foot of compressed air, assuming free air pressure of 14.5 lbs. and n equal to 1.41?

Ans. 54,936 ft.-lbs.

5. Find the theoretical horse-power developed by 3 cu. ft. of air per minute having a pressure of 200 lbs. per sq. in. absolute, being admitted and expanded in an air engine with one-fourth cut-off. The value of n is 1.2. (Neglect clearance.)

Ans. 5.01 h.p.

6. Compute the net saving in energy that is effected by compressing isothermally instead of adiabatically 50 cu. ft. of free air to a pressure of

* On the subject of transmission of power by compressed air, reference should be made to papers by Professor Richards in Bulletin No. 63 of Engineering Experiment Station of Univ. of Ill.; Weymouth on "Problems in Gas Engineering," *Trans. American Society of Mechanical Engineers*, vol. 34 (1912), pages 185-234; Baker on "Expansion and Temperature Drop of Compressed Air," *Trans. A. S. M. E.*, vol. 33, pages 918-919.

200 lbs. per sq. in. gage. Barometer = 14 lbs. per sq. in., and a temperature of 70° F. What is the increase in intrinsic energy during each kind of compression? How much heat is lost to the jacket-water during each kind of compression?

Ans. 144,000 ft.-lbs.; — 0 isothermal; 299,500 ft.-lbs. adiabatic; 275,500 ft.-lbs. isothermal; 0 adiabatic.

7. Let a volume of 12 cu. ft. of free air be adiabatically compressed in one stage from atmospheric pressure (15 lbs.) to 85 lbs. gage; the initial temperature of the air being 70° F.

(a) What is the volume and temperature of the air after the compression? *Ans.* 3.12 cu. ft.; 920° F. absolute.

(b) Suppose this heated and compressed air be cooled to an initial temperature of 60° F., what is its pressure for that condition? *Ans.* 56.65 lbs. absolute.

(c) Now if the air occupies such a volume as found in (a) and at an absolute pressure as in (b), at a temperature of 60° F., and is then allowed to expand adiabatically down to atmospheric pressure (15 lbs.), what is the temperature of the expanded air in Fahrenheit degrees? Its volume as well?

Ans. — 106.72° F.; 8 cu. ft.

CHAPTER VII

ENTROPY

PRESSURE-VOLUME diagrams are useful for determining the **work** (in foot-pounds), done during a cycle, but they are of very limited use in analyzing the heat changes involved. It has, therefore, been found desirable to make use of a diagram which shows directly by an area the number of **heat units** (instead of foot-pounds) involved during the processes constituting a cycle. In order that an area shall represent heat units instead of work

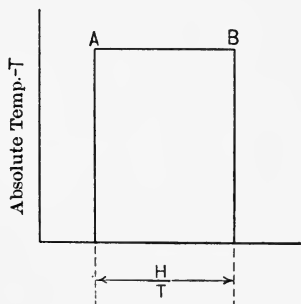


FIG. 27.—Diagram of Entropy.

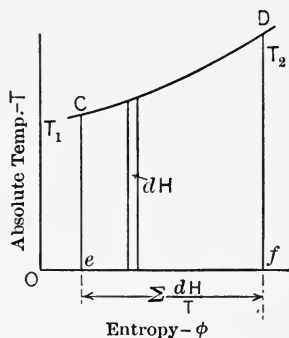


FIG. 28.—Analysis of Entropy Diagram.

units the coördinates must be such that their product will give heat units. If the ordinates are in absolute temperature, the abscissas must be **heat units per degree of absolute temperature**, that is, $\frac{H}{T}$, for then $T \times \frac{H}{T} = H$, the amount of heat added during the process from **A** to **B** (Fig. 27).

This simple ratio, heat **added** divided by the absolute temperature during addition, can be employed when the temperature remains constant; but when the temperature changes, a different form of expression must be developed. Suppose that the heat H is divided up into a number of small increments dH (Fig. 28),

and that each small increment of heat is divided by the average absolute temperature at which the heat change occurs. We will then have a series of expressions $\frac{dH}{T}$ which, when summed up, will give the total change in the abscissas. This quantity $\int \frac{dH}{T}$ when multiplied by the average absolute temperature between **C** and **D** will give the total amount of heat added during the process. Mathematically expressed, the change in the abscissas is

$$d\phi = \frac{dH}{T} \quad \text{or} \quad \phi = \int \frac{dH}{T} \quad (68)$$

and the heat change involved is

$$dH = T d\phi \quad \text{or} \quad H = \int T d\phi. \quad (69)$$

The quantity ϕ in the equations is known as the **increase in entropy** of the substance, and may be defined as a quantity which, when multiplied by the average absolute temperature occurring during a process, will give the number of heat units (in B.t.u.) added or abstracted as heat during the process. The "increase in entropy" is employed rather than entropy itself, because we are concerned only with the differences in entropy, and furthermore we could not calculate the absolute entropy, because the specific heat of a substance is not accurately known at low temperatures.

This definition of entropy means that in a diagram such as **Fig. 28**, where the ordinates are absolute temperatures, and the abscissas are entropies as calculated above some standard temperature, the area under any line **CD** gives the number of heat units added to the substance in passing from a temperature T_1 and entropy $\phi_1 = \text{Oe}$, to a temperature T_2 and entropy $\phi_2 = \text{Of}$ (or the number of heat units abstracted in passing from T_2 to T_1).

Let us apply the above conceptions of entropy to the analysis of a Carnot cycle. A pound of the working substance is expanded **isothermally**. On a T - ϕ (temperature-entropy) diagram (**Fig.**

29), this process would be represented by line **AB**, where the temperature remains constant at T_1 , and where the entropy increases from **Oe** to **Of**, because of the addition of heat that is required to keep the temperature constant. The amount of heat that is added is given by the area **ABfe**.

The next process is **adiabatic** expansion from T_1 to T_2 . Heat is neither added to nor abstracted from the substance during this expansion. Hence the entropy remains constant, as indicated by **BC**.

The substance is now isothermally compressed along **CD**, the temperature, of course, remaining constant at T_2 and the entropy decreasing because of the abstraction of heat equal to the area under **CD**, i.e., **CDef**.

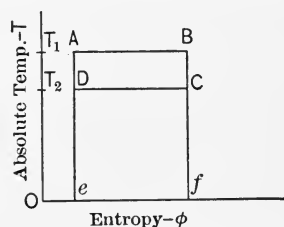


FIG. 29.—Entropy Diagram of Carnot Cycle.

The last process of the cycle is **adiabatic** compression from **D** to **A**, no heat being added or abstracted, and the entropy, therefore, remaining constant.*

To determine the amount of net work done during this cycle we can employ the familiar relation

$$(\text{Heat equivalent of}) \text{ work done} = \text{heat added} - \text{heat rejected.} \quad (70)$$

Applying equation (70) to **Fig. 29** we have

Heat added = **ABfe**; heat rejected = **CDef**. Therefore,

$$\begin{aligned} \text{Work done (in B.t.u.)} &= \text{ABfe} - \text{CDef} \\ &= \text{ABCD.} \end{aligned} \quad (71)$$

Applying the expression for efficiency, namely,

$$E = \frac{\text{work done}}{\text{heat added}},$$

we obtain then,

$$\text{Efficiency} = \frac{\text{ABCD}}{\text{ABfe}}.$$

Now the question may arise as to why the entropy remains constant during **adiabatic** expansion, as from **B** to **C**, when it is

* *Adiabatic* lines are sometimes called *isentropic* lines (lines of equal entropy).

known that at **C** there is less intrinsic energy (i.e., less heat) in the substance than at **B**, the decrease being $MC_v(T_1 - T_2)$. The answer is that while heat disappeared from the substance during the process, it disappeared as **work** and not as heat. The case of constant entropy during adiabatic expansion is thus found to be in accord with the explanation of entropy (as given on page 102), which states that the heat involved in a change of entropy must be added or abstracted as **heat**, that is, must be **conducted** or radiated to or from the substance.

From the foregoing discussion two important conclusions may be drawn in regard to the use of the T - ϕ diagram:

1. If any heat process be represented by a curve on a T - ϕ diagram, the **heat involved during the process** is equal to the area under the curve, that is, between the curve and the axis of absolute temperature.
2. If a cycle of heat processes be represented on a T - ϕ diagram by a **closed figure**, the net work done is equal to the enclosed area, that is, the enclosed area measures the amount of heat that was converted into work.

As has been stated previously, T - ϕ diagrams are very useful for analyzing heat processes, and are often referred to as "heat diagrams." They find particularly useful application in steam engineering, as indicated by the following:

1. Graphical analysis of heat transfers in a steam engine cylinder (pages 129-133).
2. Determination of quality of steam during adiabatic expansion (page 116).
3. Quick calculation of efficiency according to Rankine cycle (page 129).
4. Steam turbine calculations (page 123).

The applications of entropy to heat engineering will be readily appreciated as the subject is developed.

Temperature-Entropy Diagrams for Steam. Practically all the diagrams that have been shown in the preceding chapters indicated the relations between **pressure** and **volume**. Such diagrams show very well the action of steam in a **reciprocating**

steam engine, where steam is admitted at the beginning of a stroke, expanded and exhausted or rejected from the cylinder on the return stroke. **Steam turbines** operate differently, as there is a continual flow of steam into the high pressure end and a continual flow of low pressure steam from the exhaust. The resulting continual and uninterrupted drop in pressure and temperature cannot be satisfactorily represented with the pressure-volume diagrams that have preceded.

Another kind of diagram is, therefore, universally used by steam engineers. In this diagram, as has been stated, any surface represents accurately to given scales, a definite quantity of heat. Absolute temperatures (T) are the ordinates, and the entropies (ϕ) are the abscissas.

Figure 30 shows a simple heat diagram laid out with absolute temperature and entropy for the coördinates. Steam at a certain condition of temperature and entropy is represented here by the point **A**. Then if some heat is added, increasing both tem-

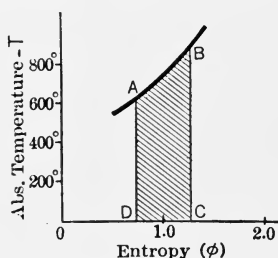


FIG. 30.—Simple Temperature-entropy Diagram of Steam.

perature and entropy, the final condition is represented by the point **B**, and the area **ABCD** represents the heat added in passing from the condition at **A** to the condition at **B**. Such a diagram is called a **temperature-entropy diagram**, although the name "heat diagram" is just as appropriate, since every area represents a definite amount of heat.

Another temperature-entropy diagram is shown in **Fig. 31** representing by the various shaded areas the heat added to water at 32°F. to completely vaporize it at the pressure P_1 . The unshaded area under the irregular curve **AA'B** represents the heat in a pound of water at the freezing point, 32°F. or 492 degrees in absolute Fahrenheit temperature. **A'B** represents the increase in entropy due to the latent heat of fusion. The area **OBCD** is the heat added to the water to bring it to the temperature of vaporization, or in other words this last area represents

additional heat required would be represented by an area to the right of **E**.

Calculation of Entropy for Steam. In order to lay off the increase in entropy as abscissas in the heat diagram for steam, it is necessary to develop formulas for calculating ϕ . For convenience 32°F. has been adopted as the arbitrary starting point for calculating increase of entropy, as well as for the other thermal properties of steam. Referring to **Fig. 32** the entropy of water is seen to be 0 at 32°F. In order to raise the temperature from T_0 to T_1 , h heat units are required, and by the equation (68), the **entropy of one pound of the liquid θ** will be

$$\theta = \int_{T_0}^{T_1} \frac{dh}{T}, \quad (72)$$

or, assuming the specific heat of water to be unity we can write

$$\begin{aligned} \theta &= \int_{T_0}^{T_1} \frac{dT}{T} \\ &= \log_e T_1 - \log_e T_0 = \log_e \frac{T_1}{T_0}. \end{aligned} \quad (73)$$

In order to evaporate the water into steam at the boiling point T_1 , the latent heat, L , must be added at constant temperature T_1 . The increase of entropy during the "steaming" process is represented by $\frac{L}{T}$ and is obviously equal to

$$\text{Entropy of Evaporation} = \frac{L_1}{T_1}. \quad (74)$$

The total entropy, ϕ , of dry saturated steam above 32°F. at temperature T_1 is, then,

$$\phi = \theta + \frac{L}{T} = \log_e \frac{T_1}{T_0} + \frac{L_1}{T_1}, \quad (75)$$

and for steam at temperature T_2 , we have

$$\phi = \log_e \frac{T_2}{T_0} + \frac{L_2}{T_2}.$$

Values of these entropies can be found in nearly all steam-tables.

Isothermal Lines of Steam. When the expansion of steam occurs at constant pressure as, for example, in the conversion of water into steam in a boiler when the engines are working, we have isothermal expansion. It must be obvious from the preceding explanation that steam (or any other **vapor**) can be expanded or compressed isothermally only when wet. Isothermal lines for wet steam, which consists of a mixture of water and its vapor, are, therefore, straight lines of uniform pressure. On a pressure-volume diagram an isothermal line is consequently represented by a horizontal line parallel to the axis of abscissas. The horizontal parts of the indicator diagram as illustrated in

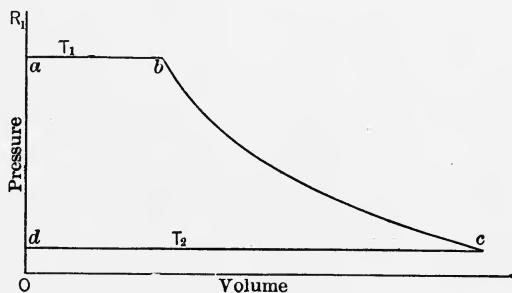


FIG. 33. — Indicator Diagram of Ideal Cycle Using Steam.

Fig. 33 are lines of constant pressure and, therefore, isothermal lines of steam. On a $T-\phi$ diagram, the isothermal line is represented by a line of constant temperature, i.e., by a line parallel to the X -axis.

Adiabatic Lines for Steam. Adiabatic lines will have different curvature as they represent expansion or compression of different substances. It will be remembered that the values of γ are different for the various gases discussed in preceding chapters, and therefore the adiabatic line for each of these gases would have a different curvature. In the same way the curvature of adiabatic lines of steam will vary with the relative amounts of steam and water in wet steam. It is worth mentioning here that steam which is initially dry, if allowed to expand adiabatically, will become wet, the percentage of moisture which it will contain depending on the extent to which the expansion is car-

ried. Also, on any T - ϕ diagram, an adiabatic (isentropic) line is represented by a line parallel to the Y -axis, i.e., by a line of constant entropy. If steam is initially wet and is expanded adiabatically, it becomes wetter as a rule.*

In general, in any sort of an expansion in order to keep steam at the same relative dryness as it was initially, while it is doing work some heat must be supplied and taken in. And if the expansion is adiabatic so that no heat is taken in, a part of the steam will be condensed and will form very small particles of water suspended in the steam, or it will be condensed as a sort of dew upon the surface of the enclosing vessel.

The relation between pressure and temperature as indicated by the steam-tables continues throughout an expansion, provided the steam is initially dry and saturated or wet.

Adiabatic Curve for Steam. Whether steam is initially dry and saturated or wet, the adiabatic curve may be represented by the formula: $PV^n = \text{constant}$. The value of the index n depends on the initial dryness of the steam. Zeuner has determined the following relation

$$n = 1.035 + \frac{x}{10}.$$

Solving this when $x = \text{unity}$ (dry and saturated steam) the value of n is 1.135, and when x is 0.75, n has the value 1.11.†

Example. One pound of steam having a quality of 0.95 at a pressure of 100 pounds per square inch absolute expands adiabatically to 15 pounds per square inch absolute.

What is the quality at the final condition?

* When the percentage of water in wet steam is very great and the steam is expanded adiabatically there is in many cases a tendency at the beginning of the expansion for the steam to become drier. This is very evident from an inspection of diagrams like Fig. 32.

† Rankine gave the value of $n = \frac{1}{9}$, which obviously from the results given is much too low if the steam is at all near the dry and saturated condition. His value would be about right for the condition when $x = 0.75$. In an actual steam engine, the expansion of steam has, however, never a close approximation to the adiabatic condition, because there is always some heat being transferred to and from the steam and the metal of the cylinder and piston.

Solution. The total entropy at the initial condition equals

$$\theta + x \frac{L}{T} = 0.4743 + 0.95 \times 1.1277 = 1.5456.$$

The total entropy at the end of the expansion equals

$$\theta + x \frac{L}{T} = 0.3133 + 1.4416 x.$$

Since entropy is constant in adiabatic expansion

$$0.3133 + 1.4416 x = 1.5456,$$

from which

$$x = 0.854.$$

How much work is done during the expansion?

Since there is no heat added the work done equals the loss in internal energy.

The internal energy at the end of the expansion equals

$$h + xI_L = 181.0 + 0.854 \times 896.8 = 946.9 \text{ B.t.u.}$$

The internal energy at the initial condition equals

$$h + xI_L = 298.3 + 0.95 \times 806.6 = 1064.6 \text{ B.t.u.}$$

The work equals

$$1064.6 - 946.9 = 117.7 \text{ B.t.u. or } 91,576 \text{ foot-pounds.}$$

Example. One pound of steam has a pressure of 100 pounds per square inch absolute and a quality of 0.95. It expands along an $n = 1$ curve to 20 pounds per square inch absolute.

What is the quality at the end of the expansion?

Solution. The volume of the steam at the initial condition is $x \times V$.

$$x \times V = 0.95 \times 4.429 = 4.207 \text{ cubic feet.}$$

Obviously, $P_1 V_1^n = P_2 V_2^n$ and since $n = 1$,

$$100 \times 4.207 = 20 \times V_2,$$

$$V_2 = 21.035 \text{ cubic feet.}$$

The volume of dry saturated steam at the end of the expansion or at 20 pounds per square inch is 20.08. Therefore the steam is superheated at end of expansion. How is this known?

From the superheated steam-tables the amount of superheat corresponding to a specific volume of 21.035 is found to be 29° F.

What is the work done during the expansion?

$$\begin{aligned}\text{Work} &= P_1 V_1 \log_e \frac{V_2}{V_1} = 100 \times 144 \times 4.207 \log_e \frac{21.035}{4.207}, \\ &= 100 \times 144 \times 4.207 \times 1.6094 \\ &= 97,499 \text{ foot-pounds or } 125.3 \text{ B.t.u.}\end{aligned}$$

How much heat must be added?

The energy of the steam at the end of the expansion equals, since the pressure is 20 pounds per square inch absolute and 29 degrees superheat:

$$1169.9 - \frac{P(V - \frac{1}{60})}{778} = 1169.9 - 77.8 = 1092.1 \text{ B.t.u.}$$

The internal energy of the steam at the initial condition is

$$h + xI_L = 298.3 + 0.95 \times 806.6 = 1064.6.$$

The internal energy of the steam was increased during the expansion by $1092.1 - 1064.6 = 27.5$. Therefore 27.5 B.t.u. of heat must be supplied to compensate this increase as well as the heat necessary to do the work. The total heat supplied to the steam during the expansion would thus be

$$\begin{aligned}27.5 + \text{work of the expansion} &= 27.5 + 125.3 \\ &= 152.8 \text{ B.t.u.}\end{aligned}$$

Example. One pound of steam at a pressure of 150 pounds per square inch absolute and a volume of 1.506 cubic feet expands under constant pressure until it becomes dry and saturated.

What is the quality at the initial condition?

Solution. The volume of dry and saturated steam at the given pressure is 3.012 cubic feet per pound.

$$\text{The quality then is } \frac{1.506}{3.012} = 0.50.$$

What is the volume of the steam at the final condition?

The volume a pound then is 3.012 cubic feet since the steam is dry and saturated. What is the work done during the expansion?

Work $P_1 (V_2 - V_1) = 150 \times 144 (3.012 - 1.506) = 32,530$
foot-pounds.

How much heat is required?

Heat added $= H_2 - (h_1 + x_1 L_1)$
 $= 1193.4 - (330.2 + 0.5 \times 863.2)$
 $= 1193.4 - 761.8 = 431.6 \text{ B.t.u.}$

PROBLEMS

1. One pound of water is raised in temperature from 60° to 90° F. What is the increase in entropy? *Ans. 0.056.*
2. Dry and saturated steam has a pressure of 100 lbs. per sq. in. absolute.
 - (a) What is the entropy of the liquid? *Ans. 0.4743.*
 - (b) What is the entropy of evaporation? *Ans. 1.1277.*
 - (c) What is the total entropy of the steam? *Ans. 1.6020.*
3. Steam at 150 lbs. per sq. in. absolute has a quality of 0.90.
 - (a) What is the entropy of the liquid? *Ans. 0.5142.*
 - (b) What is the entropy of evaporation? *Ans. 0.9490.*
 - (c) What is the total entropy of the steam? *Ans. 1.4632.*
4. Steam having a temperature of 300° F. has an entropy of evaporation of 1.1900. What is its quality? *Ans. 0.994.*
5. Steam having a pressure of 200 lbs. per sq. in. absolute has a total entropy of 1.5400.
 - (a) What is the total entropy of dry and saturated steam under the given pressure? *Ans. 1.5456.*
 - (b) Is the steam wet or dry?
 - (c) What is its quality? *Ans. 0.994.*
6. Steam having a pressure of 125 lbs. per sq. in. absolute is superheated 100° F.
 - (a) What is its total entropy? *Ans. 1.6484.*
 - (b) What is the total entropy of dry and saturated steam under the given pressure? *Ans. 1.5839.*
 - (c) What is the entropy of the superheat? *Ans. 0.0645.*
7. Steam having a pressure of 150 lbs. per sq. in. absolute has a total entropy of 1.6043.
 - (a) What is the total entropy of dry and saturated steam under the given pressure? *Ans. 1.5692.*
 - (b) Is the above steam saturated or superheated? How can you tell?
 - (c) How much superheat has the steam? *Ans. 50° F.*
8. A boiler generates steam of 0.90 quality at a temperature of 350° F. with the feed water admitted at 90° F. What is the increase in entropy? *Ans. 1.3594.*

CHAPTER VIII

PRACTICAL STEAM EXPANSIONS AND CYCLES

Efficiency of an Engine Using Steam Without Expansion. In the early history of the steam engine, nothing was known about the "expansive" power of steam. Up to the time of Watt in all steam engines the steam was admitted at full boiler pressure at the beginning of every stroke and the steam at that pressure carried the piston forward to the end of the stroke without any diminution of pressure. Under these circumstances the volume of steam used at each stroke at boiler pressure is equal to the volume swept through by the piston.

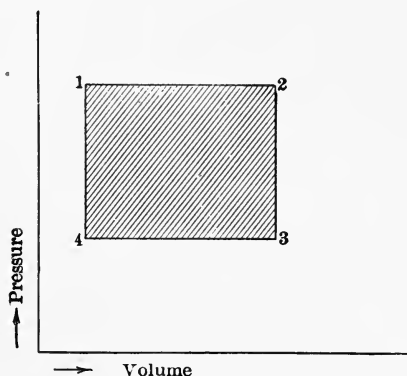


FIG. 34. — Indicator Diagram of an Engine using Steam without Expansion.

An indicator diagram representing the use of steam in an engine without expansion is shown in **Fig. 34**. This diagram represents steam being taken into the engine cylinder at 1 at the boiler pressure. It forces the piston out to the point 2 when the exhaust opens and the pressure drops rapidly from 2 to 3. On the back stroke from 3 to 4 steam is forced out of the cylinder into the exhaust pipe. At 4 the pressure rises

rapidly to that at 1 due to the rapid admission of fresh steam into the cylinder. In this case the thermal efficiency (E) is represented by

$$E = \frac{\text{work done}}{\text{heat taken in}} = \frac{(P_1 - P_2) (V_2 - V_1)}{778 (x_1 L_1 + h_1 - h_2)}, \quad (76)$$

where the denominator represents the amount of heat taken in, with the feedwater at temperature of exhaust, t_2 . In actual practice the efficiency of engines using steam without expansion is about 0.06 to 0.07, when the temperature of condensation is about 100° F. When steam is used in an engine without expansion and also without the use of a condenser the value of this efficiency is still lower. It will be observed that under the most favorable conditions obtainable the efficiency of an engine without expansion cannot be made under normal conditions to exceed about 7 per cent.

In the actual Newcomen steam engines efficiency was very much lower than any of the values given because at every stroke of the piston a very much larger amount of steam had to be taken in than that corresponding to the volume swept through by the piston on account of a considerable quantity of steam condensing on the walls of the cylinder.

Quality of Steam During Adiabatic Expansion. A very important equation, having wide application in steam engineering will now be developed. This equation is used in finding the quality of steam after adiabatic expansion, and can easily be derived after a further study of the T - ϕ diagram.

Referring to Fig. 32 (page 106), the line T_1c represents the increase of entropy due to the latent heat added during the steaming process. If this steaming process had stopped at some point such that the steam was wet, having a quality x , this condition of the steam could be denoted by the point s , where

$$x = \frac{T_1s}{T_1c}.$$

This relation is obvious, for a distance along T_1c represents the entropy of steam, which is proportional to the latent heat

added, which in turn is proportional to the amount of dry steam formed from one pound of water. In like manner, $\frac{T_2 m}{T_2 d}$ is the quality of steam that has been formed along $T_2 d$ at temperature T_2 .

It is thus apparent that any point on a T - ϕ diagram will give full information in regard to the steam. The proportional distances on a line drawn through the given point between the water and the dry steam lines and parallel to the X -axis give the quality of the steam as shown above. The ordinate of the point gives the temperature and corresponding pressure, while its position relative to other lines such as constant volume and constant total heat lines which can be drawn on the same diagram, will give further important data. Such a point (as **m**) is said to be the “**state point**” of the steam.

For the present we are particularly interested in the **quality of steam during adiabatic expansion**. Since in adiabatic expansion no heat transfer takes place, the entropy remains constant, and, therefore, on a T - ϕ diagram, this condition is represented by a straight vertical line as **cgf** (Fig. 32) or **smk**. Equation 75, (page 107), must be modified for **wet** steam as follows:

$$\phi = \theta + x \frac{L}{T} = \log_e \frac{T_1}{T_0} + x_1 \frac{L_1}{T_1}, \quad (77)$$

where x = the quality or dryness fraction of the steam, and ϕ = the total entropy of **dry saturated** steam, as before.

Since in adiabatic expansion the entropy remains constant, the following equation can be written

$$\phi_1 = \phi_2 \text{ (total entropies)}$$

or

$$\theta_1 + x_1 \frac{L_1}{T_1} = \theta_2 + x_2 \frac{L_2}{T_2}. \quad (78)$$

Knowing the initial conditions of steam, the quality of the steam at any time during adiabatic expansion can be readily determined. Thus, suppose the initial pressure of dry saturated steam to be 100 pounds per square inch absolute, and the

final pressure after adiabatic expansion 17 pounds per square inch.

From the steam-tables we find that the total entropy ϕ , for dry steam at 100 pounds pressure is 1.6020; that is,

$$\phi_1 = 1.6020.$$

The entropies at 17 pounds pressure are also obtained from the tables, and we have, substituting in equation (78)

$$1.6020 = 0.3229 + x_2 1.4215$$

whence,

$$x_2 = 0.899.$$

For a rapid and convenient means of checking the above result, the "**Total Heat-entropy**" diagram (see Marks and Davis' *Steam Tables*, Diagram I), can be used. From the intersection of the 100-pound pressure line and that of unit quality ("saturation line"), is dropped a vertical line (line of constant entropy = 1.602) to the 17-pound pressure line. This latter intersection is found to lie on the 0.90 quality line.

Graphical Determination of Quality of Steam by Throttling Calorimeter and Total Heat-entropy Diagram. It will be remembered that the throttling calorimeter (pages 74-77) depends for its action upon the fact that the **total heat** of steam which expands without doing work remains the same, the heat in excess of that required to keep the steam dry and saturated going to superheat the steam. Suppose that steam enters the calorimeter at a pressure of 150 pounds per square inch absolute, and is throttled down to 17 pounds per square inch, the actual temperature being 240° F. Since the saturation temperature for steam at 17 pounds pressure is 219.4, the steam in the calorimeter is superheated 240° - 219.4° or 20.6 degrees. In order to find the quality of the live steam refer to the "**Mollier Diagram**" (Total Heat-entropy Diagram, **Fig. 35**) and find the intersection of the 20.6 degrees superheat line with the 17-pound pressure line. From this point follow a horizontal line (line of constant total heat) to the left until it intersects the 150-pound pressure line. This point of intersection is found to lie on the 0.96 quality line.

Formula 65 (page 77) gives the following result in close agreement with the diagram:

$$x_1 = \frac{1153.1 + 0.47(240 - 219.4) - 330.2}{863.2} = 0.965.$$

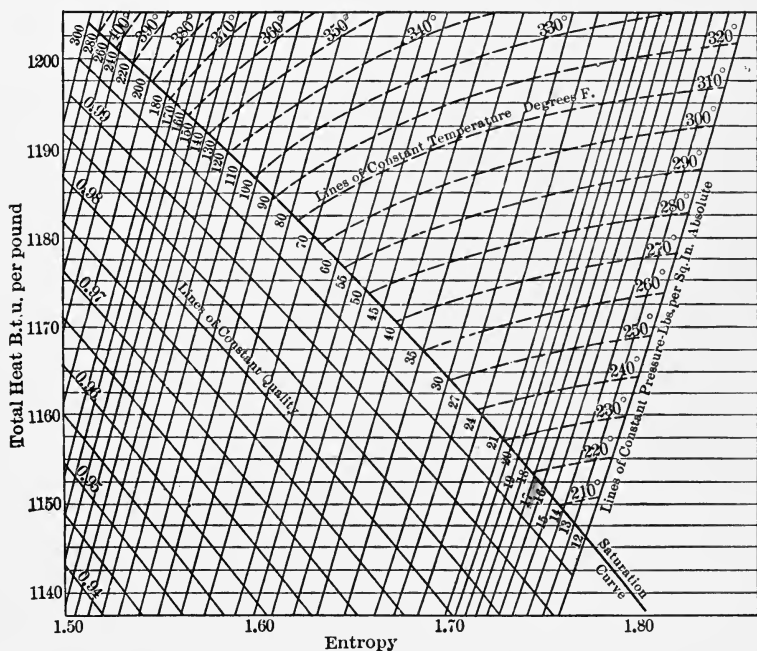


FIG. 35.—Mollier Diagram for Determining Quality of Steam.

Construction of Adiabatic Curve by Total Heat-entropy Diagram. By means of this “Mollier” diagram the curve for adiabatic expansion can be very readily drawn on a pressure-volume diagram when the initial quality of steam at “cut-off” is known. Assume that one pound of wet steam at the initial condition of pressure and quality as determined above is admitted to the engine cylinder per stroke, and that there is previously in the clearance space 0.2 cubic feet of steam (see Fig. 36), at **exactly** the same condition. The volume of a pound of dry saturated steam at this initial pressure of 150 pounds is, from

the steam-tables, 3.012 cubic feet. At 0.965 quality it will be 1 (0.965×3.012) = 2.905 cubic feet. On the scale of abscissas this amount added to the 0.20 cubic feet in clearance gives

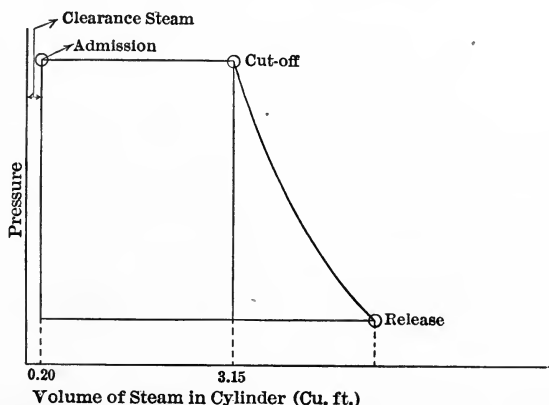


FIG. 36.—Illustrative Indicator Diagram of Engine Using Steam with Expansion.

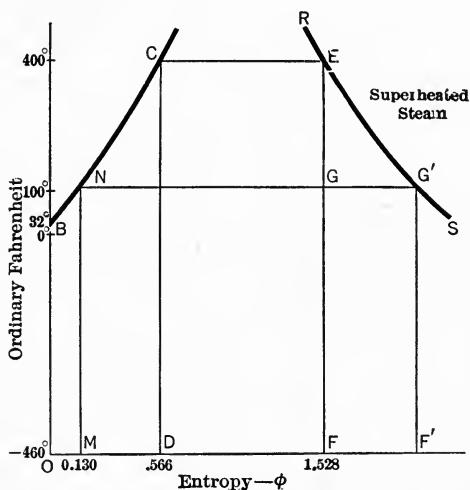


FIG. 37.—Temperature-entropy Diagram of Steam Engine.

3.105 cubic feet, the volume to be plotted at cut-off. Other points in the adiabatic expansion curve can be readily plotted after determining the quality by the method given on page 116.

The use of the temperature-entropy diagram in exhibiting the behavior of steam in an adiabatic expansion and the various heat exchanges in the passage of steam through a steam cycle will now be discussed and illustrated with a practical example.

Fig. 37 illustrates the heat process going on when feed water is received in the boilers of a power plant at 100°F. , is heated and converted into steam at a temperature of 400°F. , and then loses heat in doing work. When the feed water first enters the boiler its temperature must be raised from 100° to 400°F. before any "steaming" begins. The heat added to the liquid is the area **MNCD**. This area represents the difference between the heats of the liquid ($374 - 68$) or about 306 B.t.u. The horizontal or entropy scale shows that the difference in entropy between water at 100° and 400°F. is about 0.437.*

Every reader should understand how such a diagram is constructed and especially how the curves are obtained. In this case the curve **NC** is constructed by plotting from the steam-tables the values of the entropy of the liquid for a number of different temperatures between 100° and 400°F.

If, now, water at 400°F. is converted into steam at that temperature, the curve representing the change is necessarily a constant temperature line and therefore a horizontal, **CE**. Provided the evaporation has been complete, the heat added in the "steaming" process is the latent heat or heat of evaporation of steam (**L**) at 400°F. , which is approximately 827 B.t.u.

The change in entropy during evaporation is, then, the heat units added (827) divided by the absolute temperature at which the change occurs ($400 + 460 = 860^{\circ}\text{F. absolute}$) or

$$\frac{r}{T} = \frac{827}{860} = 0.962.$$

The total entropy of steam completely evaporated at 400°F. is, therefore, $0.566 + 0.962$, or 1.528 .† To represent this final

* As actually determined from Marks and Davis' *Steam Tables* (pages 9 and 15), the difference in entropy is $0.5663 - 0.1295$ or 0.4368 . Practically it is impossible to construct the scales in the figure very accurately.

† Entropy like the total heat (**H**), and the heat of the liquid (**h**) is measured above the condition of freezing water (32°F.).

condition of the steam, the point **E** is plotted where entropy measured on the horizontal scale is 1.528 as shown in the figure.* The area **MNCEF** represents, then, the total heat added to a pound of feed water at 100° F. to produce steam at 400° F., and then the area **OBCEF** represents, similarly, the total heat (H in the steam-tables) above 32 degrees required to form one pound of steam at 400° F.

Adiabatic Expansion and Available Energy. A practical example as to how the temperature-entropy diagram can be used to show how much work can be obtained by a theoretically perfect engine from the adiabatic expansion of a pound of steam will now be given. When steam expands adiabatically — without a gain or loss of heat by conduction — its temperature falls. Remembering that areas in the temperature-entropy diagram represent quantities of heat and that in this expansion there is no exchange of heat, it is obvious that the area under a curve of adiabatic expansion must be zero; this condition can be satisfied only by a vertical line which is a line of constant entropy.

The work done during an adiabatic process, while it cannot be obtained from a “heat diagram,” can very readily be determined from the area under the adiabatic curve of a pressure-volume diagram, or better still by the use of steam-tables as follows: In an adiabatic expansion the amount of work done is the mechanical equivalent of the loss in internal energy as explained in Chapter III. Therefore, it is only necessary to determine the internal energy of the steam at the beginning and end of the adiabatic expansion.

$$I_{1H} = h_1 + x_1 I_{1L},$$

$$I_{2H} = h_2 + x_2 I_{2L}.$$

* The point **E** is shown located on another curve **RS**, which is determined by plotting a series of points calculated the same as **E**, but for different pressures. If more heat has been added than was required for evaporation, the area **DCEF** would have been larger and **E** would have fallen to the right of **RS**, indicating by its position that the steam had been superheated. The curve **RS** is therefore a “boundary-line” between the saturated and superheated conditions. This curve can also be plotted from the values obtained from a table of the entropies of dry saturated steam.

Work during adiabatic expansion = loss in internal energy
 $= (h_1 + x_1 I_{1L} - h_2 - x_2 I_{2L}) 778$, in foot-pounds.

For the case in Fig. 37 the adiabatic expansion curve will lie along the line **EF**, and if the temperature falls to 100°F. , the expansion will be from **E** to **G**, and during this change some of the steam has been condensed. If now heat is removed from this mixture of steam and water until all the steam is reduced to the liquid state, but without further lowering of the temperature, the horizontal line **GN*** will represent the change in its condition. The quantity of heat absorbed in this last process, technically known as condensing the steam, is represented by the area **MNGF**. The difference between this heat rejected by the steam at 100 degrees and the total heat added above 100°F. is the "**available energy**" of the steam,† and is represented by the area **NCEG**. By means of diagrams like those in the preceding figures, it will now be shown how the available energy of dry saturated steam for any given conditions can be readily calculated from the data given in the steam-tables.

Fig. 38a is a temperature-entropy diagram representing dry saturated steam which is expanded adiabatically from an initial temperature T_1 , corresponding to a pressure P_1 , to a lower final temperature T_2 corresponding to a pressure P_2 . The other initial and final conditions of total heat (H) and entropy (ϕ) are represented by the same subscripts 1 and 2. The available energy or the work that can be done by a perfect engine under

* That the steam might have been dry and saturated, the expansion would have had to follow the curve **ES** and **G** would have appeared at **G'**. The heat of the liquid, h , of a pound of steam at 100°F. is represented by **OBNM**, and the heat of evaporation (L) is **MNG'F'**, so that the total heat ($h + L$ or H) is **OBNG'F'**. The total heat of wet steam is expressed by $h + xL$, where x is the quality or relative dryness. In the case of this adiabatic expansion, then, h is as before **OBNM** and xL is **MNGF**. It is obvious also that the lines **NG** and **NG'** have the same relation to each other as the areas under them, so that

$$\frac{\text{line NG}}{\text{line NG'}} = \frac{\text{area MNGF}}{\text{area MNG'F'}} = \frac{xL}{L}, \text{ or } \frac{\text{NG}}{\text{NG'}} = x \text{ (see page 118)}$$

showing that the quality of the steam at any point, **G**, on a constant temperature line is given by the ratio of **NG** to **NG'**.

† It is equal to the net work of the Rankine cycle, and depends upon the initial and final conditions of the steam.

these conditions is the area **NCEG**. It is now desired to obtain a simple equation expressing this available energy E_a in terms

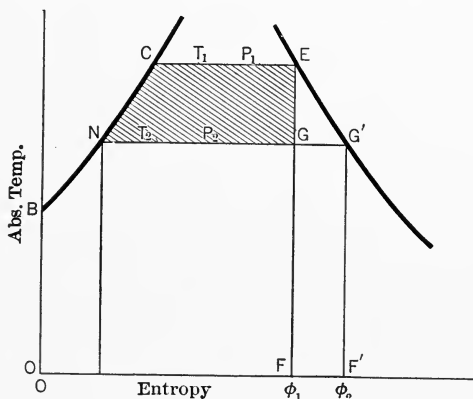


FIG. 38a. — Temperature-entropy Diagram for Dry Saturated Steam Expanded Adiabatically.

of total heat, absolute temperature and entropy. Explanations of the preceding figures should make it clear that

$$\begin{aligned}
 H_1 &= \text{area } \mathbf{OBNCEF}, \\
 H_2 &= \text{area } \mathbf{OBNG'F'}, \\
 E_a &= \text{area } (\mathbf{OBNCEF} + \mathbf{FGG'F'}) - \mathbf{OBNG'F'}, \\
 E_a &= H_1 - H_2 + (\phi_2 - \phi_1) T_2. * \quad (79)
 \end{aligned}$$

An application of this equation will be made at once to determine the heat energy available from the adiabatic expansion of a pound of dry saturated steam from an initial pressure of 165 pounds per square inch absolute to a final pressure of 15 pounds per square inch absolute.

<i>Example.</i>	$P_1 = 165$	$T_1 = 826$ degrees,	from steam-tables.
	$P_2 = 15$	$T_2 = 673.0$	" "
		$H_1 = 1195.0$ B.t.u.	" "
		$H_2 = 1150.7$ B.t.u.	" "
		$\phi_1 = 1.5615$	" "
		$\phi_2 = 1.7549$	" "

* It should be observed that this form is for the case where the steam is **initially dry and saturated**. For the case of superheated steam a slightly different form is required which is given on page 125.

Substituting these values in equation (79), we have

$$E_a = 1195.0 - 1150.7 + (1.7549 - 1.5615) 673 = 174.5 \text{ B.t.u. per pound of steam.}$$

Now if in a suitable piece of apparatus like a **steam turbine nozzle**, all this energy that is theoretically available could be changed into velocity, then we have by the well-known formula in mechanics, for unit mass,*

$$\begin{aligned} \frac{V^2}{2g} &= E_a \text{ (foot-pounds)} = E_a \text{ (B.t.u.)} \times 778, \\ V &= \sqrt{778 \times 2gE_a} = 223.8 \sqrt{E_a}, \end{aligned} \quad (80)$$

where V is the velocity of the jet and g is the acceleration due to gravity (32.2), both in feet per second.

Solving then for the theoretical velocity obtainable from the available energy we obtain the following:

$$V = 223.8 \sqrt{174.5} = 223.8 \times 13.22 = 2956 \text{ feet per second.}$$

The important condition assumed as the basis for determining equation (79), that the steam is **initially dry and saturated**, must not be overlooked in its application. There are, therefore, two other cases to be considered:

- (1) When the steam is initially wet,
- (2) When the steam is initially superheated.

Available Energy of Wet Steam. The case of initially wet steam is easily treated in the same way as dry and saturated steam. **Fig. 38b** is an example of the case in hand. At the initial pressure P_1 the total heat of a pound of wet steam ($h_1 + x_1 L_1$) is represented in this diagram by the area **OBNCE''F''**. The initial quality of the steam (x_1) is represented by the ratio of the lines $\frac{CE''}{CE}$. The available energy from adiabatic expansion from the initial temperature T_1 (corresponding to the pressure P_1) to the final temperature T_2 (corresponding to the pressure P_2) is the

* See Church's *Mechanics of Engineering*, page 672, or Jameson's *Applied Mechanics and Mechanical Engineering*, vol. I, page 47.

area $NCE''G''$. If we call this available energy E_{aw} , we have by manipulation of the areas,

$$E_{aw} = \text{area } OBNCEF + FGG'F' - OBNG'F' - G''E''EG, \\ E_{aw} = H_1 - H_2 + (\phi_2 - \phi_1)T_2 - (\phi_1 - \phi_x)(T_1 - T_2) * \quad (81)$$

or,

$$E_{aw} = H_1 - H_2 + (\phi_2 - \phi_1)T_2 - \frac{L_1}{T_1}(1 - x_1)(T_1 - T_2). \quad (82)$$

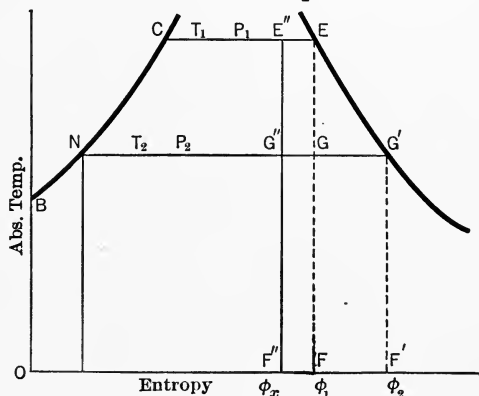


FIG. 38b. — Temperature-entropy Diagram of Wet Steam Expanded Adiabatically.

The velocity corresponding to this energy is found by substitution in equation (80), just as for the case when the steam was initially dry and saturated.

Example. Calculations for the velocity resulting from adiabatic expansion for the same conditions given in the preceding example, except that the steam is initially 5 per cent wet, are given below.

$P_1 = 165$ lbs. absolute.	$T_1 = 826$ degrees	from tables
$P_2 = 15$ lbs. absolute.	$T_2 = 673.0$ degrees	" "
	$H_1 = 1195.0$ B.t.u.	" "
	$H_2 = 1150.7$	" "
	$\phi_1 = 1.5615$	" "
	$\phi_2 = 1.7549$	" "
	$L_1 = 856.8$ B.t.u.	" "
	$x_1 = 1.00 - 0.05 = 0.95$	

$$* \phi_1 = \frac{L_1}{T_1} + \theta_1, \phi_x = x_1 \frac{L_1}{T_1} + \theta_1, \phi_1 - \phi_x = \frac{L_1}{T_1}(1 - x_1).$$

$$E_{aw} = (1195.0 - 1150.7) + (1.7549 - 1.5615) 673 - \frac{856.8}{826}$$

$$\times (1 - 0.95) (826 - 673) = 44.3 + 130.2 - 7.93$$

$$E_{aw} = 166.5 \text{ B.t.u. per pound of wet steam.}$$

$$V = 223.8 \sqrt{E_{aw}} = 223.8 \times \sqrt{166.5}$$

$$= 223.8 \times 12.9 = 2886 \text{ feet per second.}$$

This result can be checked very quickly by the "total heat-entropy" or "Mollier" diagram included in Marks and Davis' and some other steam-tables. The intersection of the 0.95 quality line and 165 pounds pressure line is found to lie on the 1152 B.t.u. total heat line. Since the expansion is adiabatic, the entropy remains constant. Therefore, following the vertical or constant entropy line (entropy = about 1.507) down to its intersection with the 15 pounds pressure line, we find that the total heat at the end of adiabatic expansion is 985 B.t.u. and

$$1152 - 985 = 167 \text{ B.t.u. available energy, as above.}$$

The velocity can be readily checked by the scale at the left of the diagram.

If the steam were superheated to begin with, the available energy during adiabatic expansion could be obtained in the same way by means of the diagram.

Available Energy of Superheated Steam. The amount of energy that becomes available in the adiabatic expansion of superheated steam is very easily expressed with the help of **Fig. 39**. Two conditions after expansion must be considered:

- (1) When the steam in the final condition is superheated,
- (2) When the steam in the final condition is wet (or dry saturated).

Using **Fig. 39** with the notation as before except E_{as} is the available energy from the adiabatic expansion of steam initially superheated in B.t.u. per pound, ϕ_s and H_s are respectively the total entropy and the total heat of the superheated steam at the initial condition, then obviously from the diagram, when the steam is wet at the final condition,

$$E_{as} = H_s - H_2 + (\phi_2 - \phi_s) T_2. \quad (83)$$

When the steam is superheated at the final condition,

$$E_{as} = H_s - H_2' - (\phi_s - \phi_2') T_2'. \quad (84)$$

It will be observed that these equations (83) and (84) are the same in form as (79), and that equation (83) differs only in having the terms H_s and ϕ_s in the place of H_1 and ϕ_1 . In other

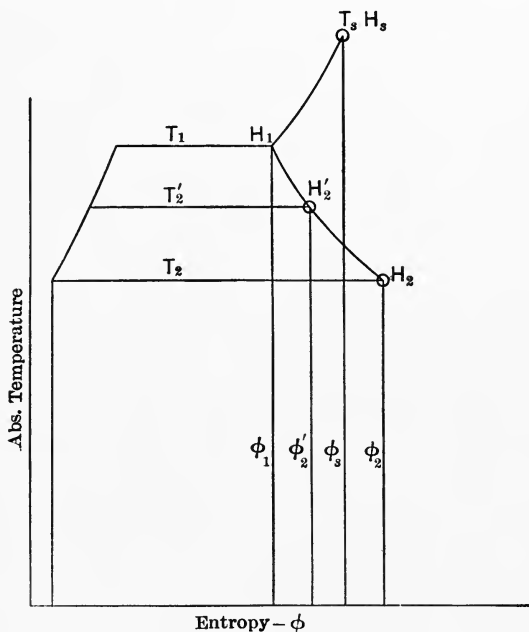


FIG. 39. —Temperature-entropy Diagram for Superheated Steam.

words equation (79) can be used for superheated steam if the total heat and entropy are read from the steam tables for the required degrees of initial superheat.

The following examples illustrate the simplicity of calculations with these equations:

Example 1. Steam at 150 pounds per square inch absolute pressure and 300° F. superheat is expanded adiabatically to 1 pound per square inch absolute pressure. How much energy in B.t.u. per pound is made available for doing work?

Solution. $H_s = 1348.8$ B.t.u. per pound,
 $H_2 = 1103.6$ " " "
 $\phi_2 = 1.980$,
 $\phi_s = 1.732$,
 $T_2 = 559.6^\circ \text{F.}$,
 $E_{as} = 1348.8 - 1103.6 + (1.980 - 1.732) 559.6$
 $= 383.9$ B.t.u. per pound.

The result above may be checked with the total heat-entropy chart in Marks and Davis' *Steam Tables and Diagrams* (Diagram I), and obtain thus (1349 - 967) or 382 B.t.u. per pound.

Example 2. Data same as in preceding example except that the final pressure is now 35 pounds per square inch absolute. (Final condition of steam is superheated.) Calculate E_{as} .

Solution. $H_s = 1348.8$ B.t.u. per pound,
 $H_2' = 1166.8$ " " "
 $\phi_s = 1.732$,
 $\phi_2' = 1.6868$,
 $T_2' = 718.9^\circ \text{F.}$,
 $E_{as} = 1348.8 - 1166.8 - (1.7320 - 1.6868) 718.9$
 $= 149.5$ B.t.u. per pound.

The Rankine Cycle.* In Chapter IV it was shown that the Carnot cycle gave the maximum efficiency obtainable for a heat engine operating between given units of temperature. In order that a steam engine may work on a Carnot cycle, the steam must be evaporated in the cylinder instead of in a separate boiler, and condensed in the cylinder, instead of being rejected to the air or to a separate condenser. Such conditions are obviously impracticable, and it has, therefore, been found necessary to adopt some other cycle which conforms more with practical conditions. The most efficient practical steam cycle, and the one which has, therefore, been adopted as the **standard** with which the efficiency of all steam engines may be compared, is the **Rankine Cycle**. The pressure-volume diagram of this cycle

* Also known as the **Clausius Cycle**, having been published simultaneously and independently by Clausius.

is shown in **Fig. 40**. Steam is admitted at constant pressure and temperature along **ab**. At **b** cut-off occurs, and the steam expands adiabatically from **b** to **c**, some of it condensing during the process. The steam is then discharged at constant pressure and temperature along the back pressure line **cd**. Line **da** represents the rise in temperature and pressure at constant volume when the inlet or admission valve opens.

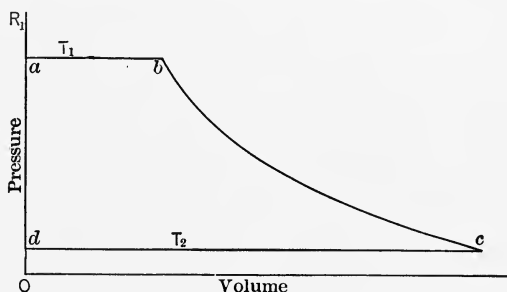


FIG. 40. — Indicator Diagram of Ideal Rankine Cycle.

The four stages of the Rankine cycle may also be stated as follows:

- (1) Feed water raised from temperature of exhaust to temperature of admission steam. (Line **da**.)
- (2) Evaporation at constant admission temperature. (Line **ab**.)
- (3) Adiabatic expansion down to back pressure. (Line **bc**.)
- (4) Rejection of steam at the constant temperature corresponding to the back pressure. (Line **cd**.)

Following the usual method for calculating the net work done in a cycle, we have, assuming one pound of steam:

$$W_{ab} = \frac{1}{778} (P_1 V_1 x_1) = \text{external work of evaporation, } E_1 \text{ (in B.t.u.),}$$

$$W_{bc} = \text{loss in internal energy} \\ = h_1 + x_1 I_{L1} - (h_2 + x_2 I_{L2}) \text{ B.t.u.,} \quad (85)$$

$$W_{dc} = -\frac{1}{778} (P_c V_c) = -\frac{1}{778} (P_2 V_2 x_2) = \text{external work of evaporation at temperature of exhaust (B.t.u.),}$$

$$W_{da} = 0.$$

Adding,

$$\text{Net work of cycle, } W = \frac{1}{778} P_1 V_1 x_1 + h_1 + x_1 I_{L1} - h_2 - x_2 I_{L2} \\ - \frac{1}{778} P_2 V_2 x_2,^*$$

but

$$\frac{1}{778} P_1 V_1 + I_{L1} = L_1 \text{ (equation (62), page 66).}$$

Therefore,

$$W = h_1 + x_1 L_1 - h_2 - x_2 L_2 \text{ (B.t.u.)}. \quad (86)$$

This means that the net work of the Rankine cycle is equal to the difference between the total heat of the steam admitted and the total heat of the steam exhausted. This statement applies whether the steam is initially wet, dry or superheated, and, therefore, it becomes a very simple matter to determine the work done in a Rankine cycle by referring to the total heat-entropy diagram.

It should be noted that this net work of the **Rankine cycle** is the so-called “**available energy**” of steam as defined and discussed in the preceding section, and that equation (82) will give the same result as equation (79). (**The student should check this statement.**)

Fig. 41 is the T - ϕ diagram for a Rankine cycle using dry saturated steam to begin with. The letters *abcd* refer to the corresponding points in the pressure-volume diagram. The net work of the cycle is **B + C**, which is the difference between the total heats at admission and exhaust. The heat added per cycle is **A + B + C + D** and the

$$\text{Thermal efficiency} = \frac{B + C}{A + B + C + D} \quad (87)$$

$$= \frac{h_1 + x_1 L_1 - h_2 - x_2 L_2}{h_1 + x_1 L_1 + h_2} \quad (88)$$

It should be carefully observed that h_2 in the denominator must always be subtracted from $h_1 + x_1 L_1$ (the total heat above 32°F.), in order to give the total heat above the temperature of

* The final quality can be determined by equation (78).

feed water, which in engine tests is always assumed for the purpose of comparison to be the same as the exhaust temperature.

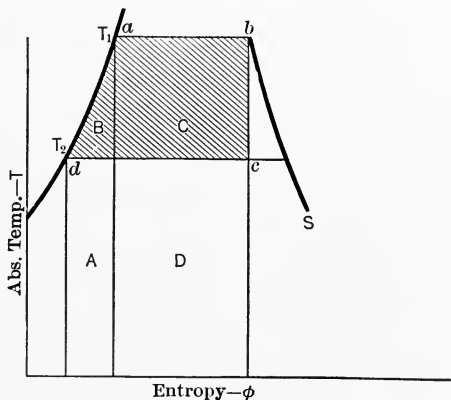


FIG. 41. — Temperature-entropy Diagram of Rankine Cycle.

One pound of steam at a pressure of 160 lbs. per sq. in. absolute and quality of 0.95 performs a Rankine cycle exhausting at 5 lbs. per sq. in. absolute.

What is the quality of the exhaust ?

Solution. The total entropy at the initial condition

$$= .5208 + .95 \times 1.0431.$$

The total entropy at the exhaust

$$= .2348 + 1.6084x.$$

Then $.5208 + .95 \times 1.0431 = .2348 + 1.6084x$.

From which $x = 0.794$.

What is the net work of the cycle ?

$$\begin{aligned} \text{Work} = H_1 - H_2 &= 335.6 + .95 \times 858.8 - 130.1 - .794 \\ &\times 1000.3 = 227.2 \text{ B.t.u.} \end{aligned}$$

What is the efficiency of the cycle ?

$$\text{Efficiency} = \frac{227.2}{335.6 + .95 \times 858.8 - 130.1} = .222 \text{ or } 22.2 \text{ per cent.}$$

The Practical or Actual Steam Engine Cycle. In the steam engine designed for practical operation it is impossible to expand

the steam down to the back-pressure line; and, furthermore, it is evident that some mechanical clearance must be provided. The result is that in the indicator diagram from an actual steam engine, we have to deal with a clearance volume, and both incomplete expansion and incomplete compression as shown in **Fig. 42**. In order to calculate the theoretical efficiency of this prac-

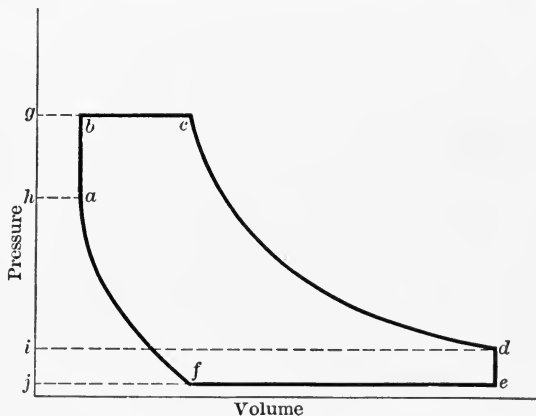


FIG. 42. — Indicator Diagram of Practical Engine Cycle.

tical cycle, it is necessary to assume that the expansion line **cd** and the compression line **fa** are adiabatic. Knowing then the cylinder feed of steam per stroke and the pressure and volume relations as determined from the indicator diagram, one can calculate the theoretical thermal efficiency by obtaining the net area of the diagram (expressed in B.t.u.) and dividing by the heat supplied per cycle. In order to obtain the net area of the diagram, the latter may be divided up into several simple parts as follows:

- gc di** — a Rankine cycle,
- ide j** — a rectangle,
- haf j** — a Rankine cycle (negative),
- gb ah** — a rectangle (negative).

The above areas can be evaluated in B.t.u. by methods previously explained (see pages 113 and 129), and thus the net work of the cycle can be determined.

The heat added per cycle is equal to
 Total heat of cylinder feed (b to c) — cylinder feed \times heat of liquid of feed water at the temperature of the exhaust.

Exercise. Calculate the thermal efficiency for the "practical" indicator diagram shown in Fig. 42, having given the following data:

- Cylinder feed = 1 cu. ft.,
 Initial steam pressure = 165 lbs. per sq. in. absolute (dry saturated),
 Exhaust steam pressure = 2 lbs. per sq. in. absolute.

PROBLEMS

1. What is the entropy of the liquid of steam of 92 per cent quality at a pressure of 15 lbs. per sq. in. absolute? *Ans.* 0.313.

2. With a quality of 0.90, what is the entropy of evaporation of steam at a pressure of 25 lbs. per sq. in. absolute? *Ans.* 1.224.

3. If there is 1.2 lbs. of water in 8 lbs. of wet steam, what is its quality? *Ans.* 0.85.

4. What is the total entropy of steam of 94 per cent quality at a pressure of 100 lbs. per sq. in. absolute? *Ans.* 1.534.

5. It was found that an engine operating under a pressure of 155 lbs. per sq. in. absolute was using steam at a temperature of 561° F. What was the condition of the steam? *Ans.* Superheat 200° F.

6. If steam at 200 lbs. per sq. in. absolute, 95 per cent quality, is caused to expand adiabatically to 228° F., what are the properties at the lower point? (That is, final total entropy, entropy of evaporation, quality and volume.) *Ans.* 1.495; 1.1600; 0.831; 16.7 cu. ft.

7. One pound of steam at a pressure of 100 lbs. per sq. in. absolute and a quality of 50 per cent is expanded isothermally until it is dry and saturated. Find the heat supplied and the work done.

Ans. 444.0 B.t.u. and 31,890 ft.-lbs.

8. What will be the final total heat of dry saturated steam that is expanded adiabatically from 150 lbs. per sq. in. absolute down to 10 lbs. per sq. in. absolute? *Ans.* 999.7 B.t.u.

9. Steam having a quality of 20 per cent is compressed along an adiabatic curve from a pressure of 20 lbs. per sq. in. absolute to a pressure whose temperature is 293° F. What is the final quality? *Ans.* 15.4 per cent.

10. Determine the final quality of the steam and find the quantity of work performed by 2 lbs. of steam in expanding adiabatically from 250 lbs. per sq. in. absolute pressure to 100 lbs. per sq. in. absolute, the steam being initially dry and saturated. *Ans.* 93.4 per cent; 129.48 B.t.u.

11. Assume that 1 lb. of steam of a pressure of 160 lbs. per sq. in. absolute and a quality of 95 per cent performs a Rankine cycle, being exhausted at a pressure of 5 lbs. per sq. in. absolute. Compute the quality of the steam exhausted and the efficiency of the cycle. Find the final volume.

Ans. 79.4 per cent; 22.2 per cent; 58.2 cu. ft.

12. What is the work of a Rankine cycle if the steam initially at 200 lbs. per sq. in. absolute pressure, superheated 200° F., goes through such a cycle with a back pressure of 1 lb. per sq. in. absolute? If, instead of expansion in such a cycle, the steam (for same conditions) were to expand in a turbine nozzle, what would be the velocity of the steam leaving?

Ans. 385 B.t.u.; 4390 ft. per sec.

13. One pound of the steam at a pressure of 100 lbs. per sq. in. absolute with a quality of 0.90 performs a Rankine cycle exhausting at a back pressure of 2 lbs. per sq. in. absolute.

What is the net work of the cycle?

Ans. 235 B.t.u.

What is the efficiency of the cycle?

Ans. 23.4 per cent.

14. Two pounds of steam at a pressure of 125 lbs. per sq. in. absolute and a volume of 8.34 cu. ft. performs a Rankine cycle. The exhaust pressure is 25 lbs. per sq. in. absolute.

What is the net work of the cycle?

Ans. 260 B.t.u.

What is the efficiency of the cycle?

Ans. 12.5 per cent.

Application of Temperature-entropy Diagram to Analysis of Steam Engine. The working conditions of a steam engine, as stated before, can be shown not only by the indicator card, but also by the employment of what is known as a "temperature-entropy" diagram. These diagrams represent graphically the amount of heat actually transformed into work, and in addition, the distribution of losses in the steam engine.

For illustration, a card was taken from a Corliss steam engine having a cylinder volume of 1.325 cubic feet, with a clearance volume of 7.74 per cent, or 0.103 cubic feet; the weight of steam in pounds per stroke (cylinder feed plus clearance) was 0.14664 pounds. Barometer registered atmospheric condition as 14.5 pounds per square inch. The scale of the indicator spring used in getting the card was 80 pounds to the inch. Steam chest pressure was taken as 153 pounds per square inch (absolute), and a calorimeter determination showed the steam to be practically dry and saturated.

The preliminary work in transferring the indicator card to a T - ϕ diagram, consists first in preparing the indicator card. It was divided into horizontal strips at pressure intervals of 10 pounds with the absolute zero line taken as a reference; this line was laid off 14.5 pounds below atmospheric conditions. (See Fig. 43a.) For reference, the saturation curve was drawn. Having known the weight of steam consumed per stroke and specific volume of steam (from the Steam Tables), for various pressures taken from the card, the corresponding actual volumes could be obtained; this operation is, merely, weight of steam per stroke multiplied by specific volume for some pressure ($0.14664 \times$ column 5 in the table below), the resulting value being the volume in cubic feet for that condition. These pressures and volumes were plotted on the card and the points joined, resulting in the saturation curve, 2''-6''-8''-9''.

The next step consisted in constructing a "transformation" table with the columns headed as shown. All the condensing and evaporation processes are assumed to take place in the cylinder and the T - ϕ diagram is then worked up for a total weight of **one pound** of steam as is customary. Column 1 shows the respective point numbers that were noted on the card; column 2, the absolute pressures for such points; column 3, the corresponding temperatures for such pressures; column 4, the volume in cubic feet up to the particular point measured from the reference line of volumes; column 5, the specific volume of a pound of dry and saturated steam at the particular pressure (Steam Tables); column 6, the volume of actual steam per pound, obtained by dividing the volumes in column 4 by 0.14664 pound (total weight of steam in cylinder per stroke); column 7, the dryness factor " x ," found by dividing column 6 by column 5; column 8 is the entropy of evaporation $\left(\frac{L}{T}\right)$ for particular conditions (Steam Tables); column 9 is the product of column 7 and column 8; column 10 is the entropy of the liquid at various conditions as found in the Steam Tables; column 11 is the sum of column 9 and column 10, giving the total entropy.

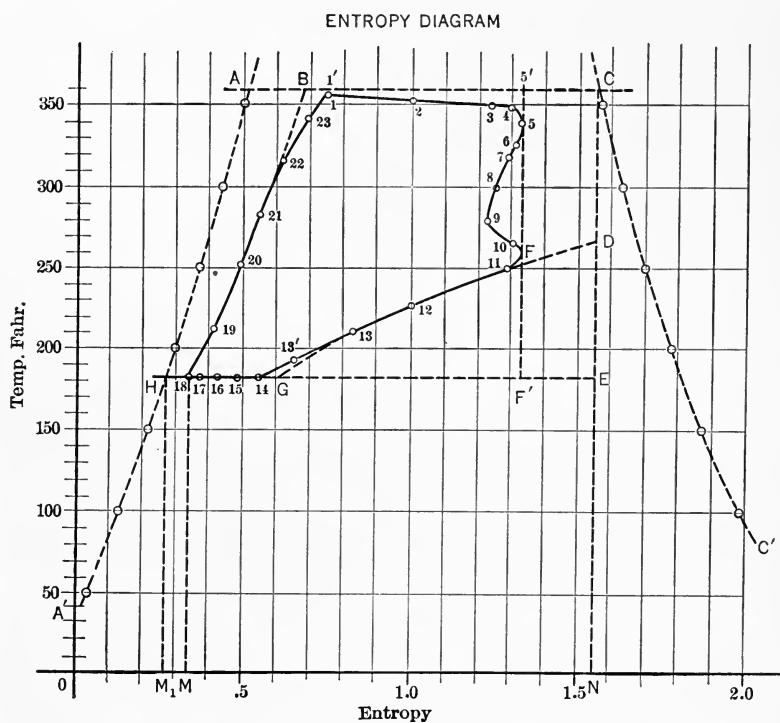
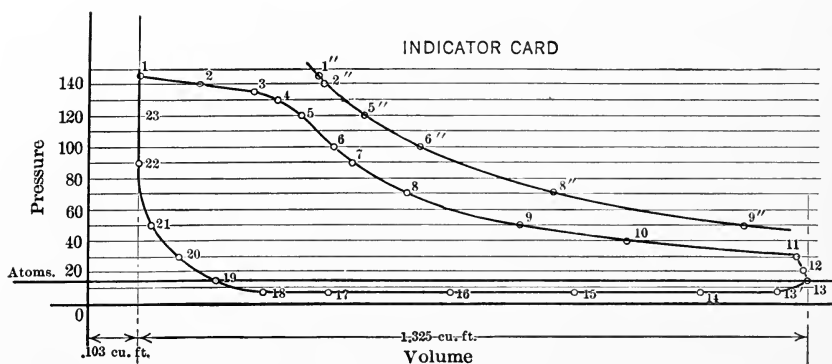


FIG. 43a.—Temperature-entropy Diagram of Actual Steam Engine Indicator Diagram.

TRANSFORMATION TABLE

1	2	3	4	5	6	7	8	9	10	11
Point number.	Pressure, lbs. per sq. in. abs.	Temperature. (Fahr.)	Volume to a certain point.	Sp. vol. per lb. dry and saturated steam.	Volume of actual steam per lb. (cu. ft.)	Dryness fraction (x).	Entropy of evaporation. L/T	$\frac{L}{T} + \frac{1}{T}$	Entropy liquid. (θ)	Total entropy. (ϕ)
1	145	356	0.1050	3.11	0.716	0.2302	1.0612	0.2440	0.5107	0.7547
2	140	353	0.2250	3.22	1.536	0.4770	1.0675	0.5090	0.5072	1.0162
5	120	341	0.4230	3.73	2.890	0.7740	1.0954	0.8475	0.4919	1.3394
9	48	279	0.8500	8.84	5.810	0.6580	1.2536	0.8250	0.4077	1.2327
12	20	228	1.4225	20.08	9.700	0.4830	1.3965	0.6740	0.3355	1.0095
15	8	183	0.9758	47.27	6.660	0.1410	1.5380	0.2168	0.2673	0.4841
18	8	183	0.3500	47.27	2.390	0.0506	1.5380	0.0778	0.2637	0.3451
20	30	250	0.1825	13.74	1.245	0.0907	1.3311	0.1208	0.3680	0.4888

Above table is employed for transferring the P - V diagram to the T - ϕ diagram.

After this table was completed, columns 3 and 11 were plotted. Convenient scales were selected, the ordinates as temperatures and the abscissas as entropies. The various points, properly designated, were connected as shown on the T - ϕ diagram, the closed diagram resulting. This area shows the amount of heat actually transformed into work. This diagram is the actual temperature-entropy diagram for the card taken and may be superimposed upon the Rankine cycle diagram in order to determine the amount and distribution of heat losses.

The water line, A - A' , and the dry steam line, C - C' , were drawn directly by the aid of Steam Table data, i.e., the entropy of the liquid and the entropy of the steam taken at various temperatures, and plotted accordingly.

Before the figure could be studied to any extent, the theoretical (Rankine) diagram had to be plotted, assuming that the steam reaches cut-off under steam-chest conditions; that it then expands adiabatically down to back pressure and finally exhausts at constant pressure to the end of the stroke without compression. This diagram is marked, A - C - E - H , on the T - ϕ plane. The steam-chest pressure of 153 pounds per square inch absolute

fixes the point, C , when the temperature line cuts the steam line, $C-C'$. The rest of the cycle is self-evident.

Referring to the $T-\phi$ diagram, **Fig. 43a**, $H-A-C-E$ is the Rankine operation with no clearance for one pound of working fluid. The amount of heat supplied is shown by the area, $M_1-H-A-C-N$, and of this quantity, the area $M_1-H-E-N^*$ would be lost in the exhaust while the remainder, $H-A-C-E$, would go into work. This is theoretical, but in practice there are losses, and for that reason, the Rankine cycle is used merely for comparison with the actual card as taken from a test. The enclosed irregular area, 1-2-3 . . . 22-23, is the amount of heat going into **actual work**. By observation, it is evident that a big area remains; this must represent losses of some sort or other. That quantity of work represented by the area, 1-5-5'-1', is lost on account of wire-drawing; the area 5'- C - D - F , shows a loss due to initial condensation; the loss due to early release is shown by the area F -12-14- F' for the real card, and by $D-G-E$ for the modified Rankine cycle (such a loss, in other words, is due to incomplete expansion); that quantity represented by 22- B -1'-1 is lost on account of incomplete compression, and $H-A-B$ -18 is the loss due to clearance. The expansion line from 5 on to 9 indicates that there is a loss of heat to the cylinder walls, causing a loss of entropy; from 9 on to 10, re-evaporation is taking place (showing a gain of entropy).

All of the heat losses are not necessarily due to the transfer of heat to or from the steam, as there may be some loss of steam due to leakage. In general, however, the $T-\phi$ diagram is satisfactory in showing heat losses.

Fig. 43a was constructed for the purpose of showing how the **actual thermal efficiency** and the **theoretical thermal efficiency** (based on the Rankine cycle) can be obtained from the $T-\phi$ diagram. The letters in **Fig. 43a** refer to the same points as in **Fig. 43b**, the only difference between the two diagrams

* The areas M_1HACN and M_1HEN should have added to them, the rectangular area between M_1-N , and the absolute zero line (**Fig. 216**), which is not shown on the diagram (**Fig. 43a**).

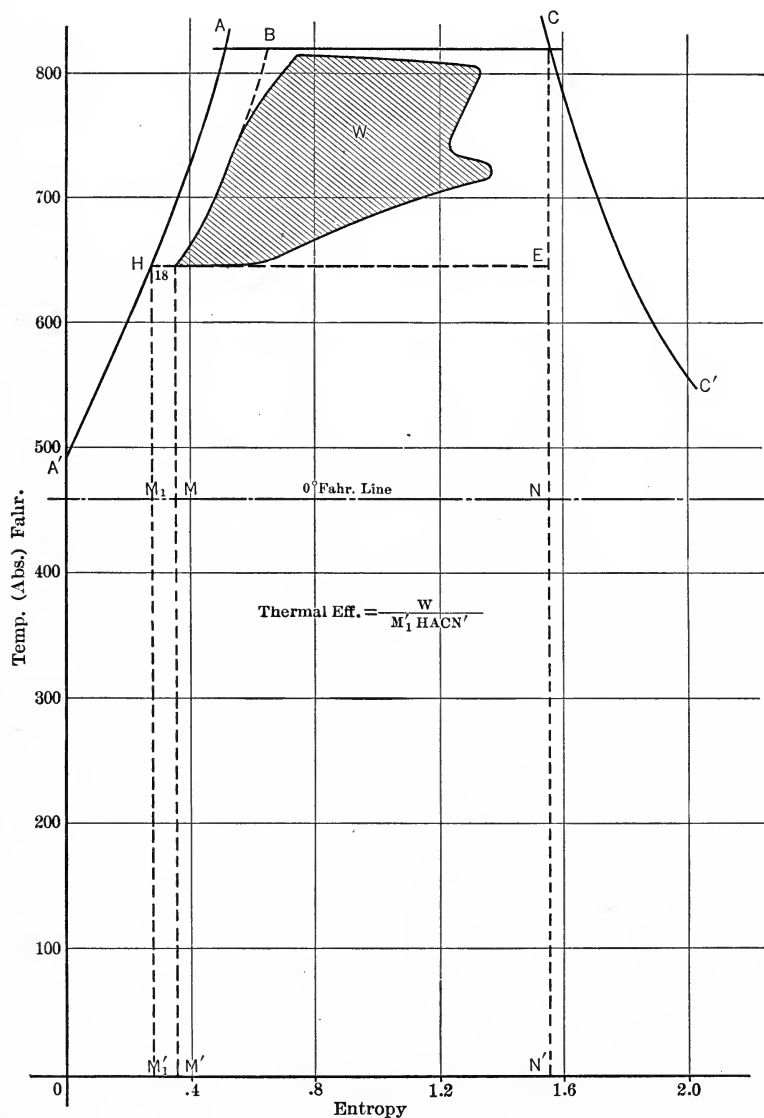


FIG. 43b.—Temperature (absolute) -entropy Diagram of Actual Steam Engine Indicator Diagram.

being the addition of the absolute zero temperature line to Fig. 43b.

$$\text{Thermal efficiency} = \frac{\text{Work done}}{\text{Heat added}}$$

$$\begin{aligned}\text{Actual thermal efficiency} &= \frac{\text{Shaded area } W}{M_1'H-A-C-N'}, \\ &= \frac{2.50 \text{ sq. in.}}{26.00 \text{ sq. in.}} \text{ (Planimeter)} \\ &= 0.096, \text{ or } 9.6 \text{ per cent.}^*\end{aligned}$$

$$\begin{aligned}\text{Rankine cycle efficiency (theoretical thermal efficiency)} &= \frac{H-A-C-E}{M_1'H-A-C-N'}, \\ &= \frac{5.29}{26.00} \text{ (Planimeter)} \\ &= 0.203, \text{ or } 20.3 \text{ per cent.}\end{aligned}$$

Discussion of Heat Engine Efficiencies. The action of all heat engines is, in general, to produce motion against a resistance or to perform work, that is to say, engines or machines by whose agency one form of energy is converted into an energy of another kind. The heat engine may be a motive power engine, in which energy in the form of heat is converted into energy in the form of mechanical work; or it may be one in which mechanical energy is transformed into the potential energy of a gas.

All these engines depend for their action on the variation in volume, temperature and pressure of some gas or vapor, or upon the explosion of some gaseous mixture. The gas or vapor which undergoes these changes in a heat engine is called the **working fluid**. In an air compressor, the working fluid is air. In the gas engine, work is done by the expansion of the products of com-

* This value may be regarded as the actual thermal efficiency for *one stroke*, inasmuch as in the succeeding strokes the "cushion" steam will be used over and over again and hence will not constitute a heat loss. The actual thermal efficiency of a steam engine under running conditions would then be given by $\frac{W}{M' \text{ 18 } BCE}$ where 18 BCE is a Rankine cycle with clearance and complete compression.

bustion of a mixture of air and combustible gas. In the oil engine, the action is the same except that oil is used. The steam engine, the engine with which we are mostly concerned, uses the vapor of water for its working fluid.

In all these heat engines the working fluid undergoes a regular series of changes, the same change occurring in the same order, over and over again. So, we can say, the working fluid goes through a circuit or "**cycle**" of operation. Beginning at a particular condition it passes through a series of successive states of pressure, volume and temperature and returns to the initial condition. Thus, the engine has started from a certain point, has gone through a regular series of movements in a fixed order, and has returned to the place from which it started. Such a series of regular changes is called a **cycle**.

The simplest form of reversible cycle is the Carnot wherein the working fluid undergoes four reversible processes between the temperatures T_1 and T_2 (page 45).

All heat engines work between some range of temperature, as in this case, T_1 to T_2 , and in view of this fact, the available relation of heat is, as explained in Chapter III,

$$E_1 = \frac{T_1 - T_2}{T_1}. \quad (89)$$

The heat represented by the temperature range between T_1 and T_2 is the only heat which under any consideration can be turned into work. It represents the "**maximum possible efficiency**" (E_1). In fact, an **ideal** steam engine is impossible. Such an engine with this cycle cannot be actually constructed because of the various thermodynamic losses.

The above expression, however, represents the efficiency of the Carnot cycle and merely signifies that it is the highest value that is obtainable for such a cycle working between such temperature limits.

The amount of work performed by the working fluid upon the piston of an engine is obviously equal to the mechanical equivalent of the difference between the **heat supplied** (H_1) and the

heat abstracted (H_2); and the **theoretical thermal efficiency** (E_2), of the cycle is therefore given by the expression,

$$E_2 = \frac{H_1 - H_2}{H_1}. \quad (90)$$

We know that E_1 is the highest possible efficiency that is attainable. E_2 , the theoretical thermal efficiency of the cycle, must then approach E_1 as its limit. So the ratio of the one to the other gives what is known as the “**type efficiency**” (E_3).

$$E_3 = \frac{E_2}{E_1}. \quad (91)$$

The closer the type efficiency approaches unity, the better the cycle should be.

It is a known fact that steam engines have very large heat losses, — losses that cannot be entirely removed. If for a certain amount of work (W) actually accomplished (equivalent heat units), the amount of heat used in the production of that work is H_a , then the “**actual thermal efficiency**” (E_4) is stated as

$$E_4 = \frac{W}{H_a}. \quad (92)$$

Theory implies that there is an efficiency that the actual thermal efficiency must approach as a limit. This relation between the actual thermal efficiency (E_4) and the theoretical thermal efficiency (E_2) is known as the **practical efficiency** (E_5) and shows how nearly the theoretical efficiency is approached, by the actual efficiency, or

$$E_5 = \frac{E_4}{E_2}. \quad (93)$$

Most important of all efficiencies is that for the “**mechanical efficiency**,” meaning the comparison of the useful work performed with the amount of work theoretically possible to obtain with a perfect machine. In other words, in a heat engine the

mechanical efficiency (E_6), is the ratio of the brake horse power to the indicated horse power, or

$$E_6 = \frac{\text{b.h.p.}}{\text{i.h.p.}}. \quad (94)$$

Another efficiency often applied is what is known as the "overall efficiency" (E_7), and is the relation of the output, or the useful work performed, divided by the heat supplied. Such an efficiency can be expressed in various ways, as

$$\begin{aligned} E_7 &= E_1 \times E_3 \times E_5 \times E_6, \\ &= E_2 \times E_5 \times E_6, \\ &= E_4 \times E_6. \end{aligned}$$

It is noted that the "overall efficiency" is the product of several efficiencies, and in view of that fact, the increasing of any of the other six respective efficiencies means a relative increase in the "overall efficiency."

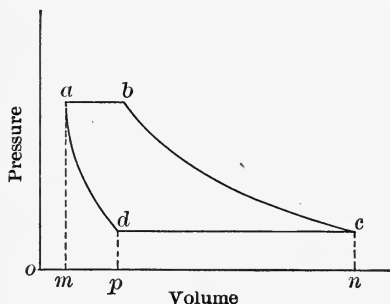


FIG. 44a. — Pressure-volume Diagram of Carnot Cycle of Steam Engine.

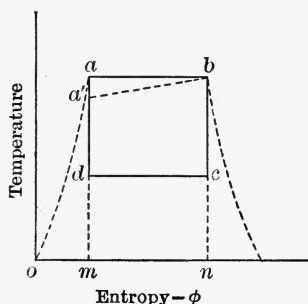


FIG. 44b. — Temperature-entropy Diagram of Carnot Cycle of Steam Engine.

Referring to the Carnot cycle for a steam engine (Fig. 44a), the amount of work (W) actually done is shown by the area **abcd** on the pressure-volume diagram and also on the temperature-entropy diagram (Fig. 44b). Of course the area **mabcn** (Fig. 44a) represents the maximum work possible with the heat (H_1) that was supplied, while **madcn** represents the amount of heat (H_2) that was abstracted from the cycle. This abstraction of

heat is a loss, so that the net work done is the difference between **mabcn** and **mdcn**, or area **abcd**. The same is expressed by symbols of heat units as $H_1 - H_2$.

Taking the Carnot cycle on the temperature-entropy diagram (**Fig. 44b**), the efficiency (E_1) is

$$E_1 = \frac{\text{abcd}}{\text{mabn}}, \text{ which equals } \frac{H_1 - H_2}{H_1}.$$

If in this cycle heat was not added along the admission line **ab** but along some other line such as **a'b**, the heat added would be decreased by the triangular area **a'ab**, and consequently the work done would be correspondingly reduced. Hence in this case, the efficiency of the modified cycle (E_1') is

$$E_1' = \frac{\text{abcd} - \text{a'ab}}{\text{mabn} - \text{a'ab}}.$$

Such a decrease in the heat supplied means an actual decrease in efficiency. From this analysis, one can say that for the obtaining of the maximum efficiency (E_1), heat must either be added or subtracted along a constant temperature line, such as **ab** and not **a'b**, and T_1 and T_2 which govern the range of temperatures should be separated as widely as possible. This statement must not be interpreted, however, to mean that when heat is added with a varying temperature (finally, attaining a higher value), that it may not be more efficient than one in which heat is added at a constant temperature of lower value. Cycles of such kinds are numerous. From an inspection of **Fig. 45** it can be seen that **abcd** is more efficient than **mnop**. Hence the greatest efficiency is obtained when all the heat is added at the highest possible temperature and all the heat removed at the lowest possible. This is true for all cycles.

Thus in **Fig. 46**, with saturated steam, the efficiency of the steam engine having such a temperature-entropy diagram, would be $\frac{\text{abcd}}{\text{fdabg}}$; and with superheated steam, a slight increase in efficiency is observed, as expressed by $\frac{\text{abb'c'd}}{\text{fdabb'h}}$. The relations,

$\frac{abcd}{fdabg}$ and $\frac{aec'd}{fdaeh}$, are practically equal. Now, if the little area **beb'** is added to both numerator and denominator of the last expression the efficiency for the superheated cycle is found. This adding of the triangle **beb'** has the same relative effect as the

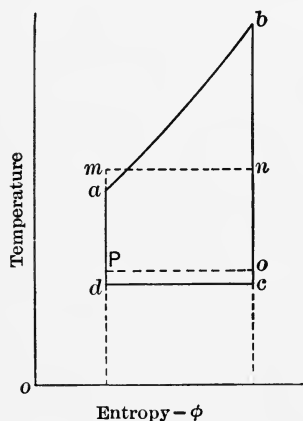


FIG. 45.—Temperature-entropy Diagram Showing Heat Added at Increasing Temperature.

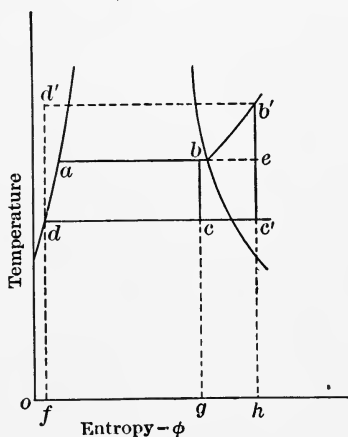


FIG. 46.—Temperature-entropy Diagram for Engine Using Superheated Steam.

subtracting of the triangle **a'ab** in the Carnot cycle (Fig. 44b). In the latter case the efficiency was increased while in the former it was diminished. Hence we can say that **superheated steam will increase the theoretical thermal efficiency of the engine.**

Various explanatory solutions for efficiencies of thermal machinery, such as steam engines, steam turbines, "locomobiles" units, gas producers and engines, blast-furnace gas engines, oil engines and pumping engines will now be given.

I. Simple Non-condensing Steam Engine. An engine developing 135 i.h.p. and using dry saturated steam at 125 pounds per square inch absolute, running with a back pressure of 0.2 pound per square inch gage and with a barometer pressure equal to 14.8 pounds per square inch, uses 32 pounds of steam per i.h.p. per hour. At end of expansion, the pressure is 20 pounds per square inch gage. The brake horse power is 125.

Find the various efficiencies (E_1 , E_2 , E_3 , E_4 , E_5 , E_6 and E_7) and also the number of thermal units consumed per i.h.p. per minute.

(a) Maximum Possible Efficiency (E_1).

$E_1 = \frac{T_1 - T_2}{T_1}$, where T_1 is the absolute temperature of the steam at the initial pressure condition, and T_2 is the absolute temperature of the steam at atmospheric.

$$\begin{aligned} E_1 &= \frac{(344.4 + 460) - (213 + 460)}{344.4 + 460} \\ &= \frac{131.4}{804.4} = 0.1614 \text{ or } 16.14 \text{ per cent.} \end{aligned}$$

(b) Theoretical Thermal Efficiency (E_2).

$$\begin{aligned} E_2 &= \frac{H_1 - H_2}{H_1} \\ &= \frac{(h_1 + x_1 L_1) - (h_2 + x_2 L_2) + \frac{1}{7} \frac{4}{8} (P_4 - P_5) \times x_2 V_4}{(h_1 + x_1 L_1) - h_5} \end{aligned}$$

The quantity $(h_1 + x_1 L_1)$, the total heat, is read from the "Mollier Diagram,"* the steam being at an initial condition of 125 pounds per square inch absolute with a quality x_1 of unity. From this point the expansion takes place adiabatically to the end of expansion at 34.8 pounds per square inch absolute, where the quality x_2 is found to be 0.9225 and the total heat $(h_2 + x_2 L_2)$ is 1093 B.t.u.; P_4 is the pressure at end of expansion, P_5 is absolute back pressure, V_4 is the volume corresponding to the pressure P_4 (from *Steam Tables*) and h_5 , the heat of the liquid at the back pressure (14.8 + 0.2) or 15 pounds per square inch absolute, then

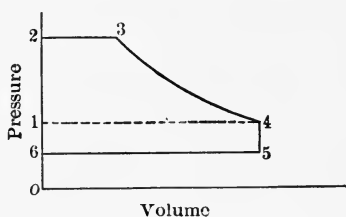


FIG. 48. Diagram of Steam Engine Cycle.

* Diagram I, Marks and Davis' *Steam Tables and Diagrams*.

$$\begin{aligned}
 E_2 &= \frac{1189 - 1093 + \frac{144}{778} \times (29.8 - 20) \times 0.9225 \times 11.89}{1189 - 181}, \\
 &= \frac{1189 - 1093 + 19.85}{1189 - 181}, \\
 &= \frac{115.85}{1008} = 0.115 \text{ or } 11.5 \text{ per cent.}
 \end{aligned}$$

(c) Type Efficiency (E_3).

$$\begin{aligned}
 E_3 &= \frac{E_2}{E_1}, \\
 &= \frac{11.5}{16.14} = 0.713 \text{ or } 71.3 \text{ per cent.}
 \end{aligned}$$

(d) Actual Thermal Efficiency (E_4).

$$E_4 = \frac{W}{H_a},$$

where (W) is the amount of work actually accomplished per hour and (H_a) is the amount of heat used in developing that work, then

$$\begin{aligned}
 E_4 &= \frac{2545^*}{32 (1189 - 181)}, \\
 &= \frac{2545}{32 \times 1008} = 0.0788 \text{ or } 7.88 \text{ per cent.}
 \end{aligned}$$

(e) Practical Efficiency (E_5).

$$\begin{aligned}
 E_5 &= \frac{E_4}{E_2}, \\
 &= \frac{7.88}{11.5} = 0.686 \text{ or } 68.6 \text{ per cent.}
 \end{aligned}$$

(f) Mechanical Efficiency (E_6).

$$\begin{aligned}
 E_6 &= \frac{\text{b.h.p.}}{\text{i.h.p.}}, \\
 &= \frac{125}{135} = 0.926 \text{ or } 92.6 \text{ per cent.}
 \end{aligned}$$

* Both numerator and denominator will be expressed in terms of B.t.u. per hour. The numerator is the heat equivalent of one horse-power-hour or $(33,000 \times 60) \div 778 = 2545$ B.t.u. per hour. The denominator is the net heat used at the rate of 32 pounds of steam per hour.

(g) Overall Efficiency (E_7).

$$\begin{aligned} E_7 &= E_4 \times E_6 \\ &= 0.0788 \times 0.926 \\ &= 0.073 \text{ or } 7.3 \text{ per cent.} \end{aligned}$$

For a check, by using another formula, we have

$$\begin{aligned} E_7 &= E_2 \times E_5 \times E_6 \\ &= 0.115 \times 0.686 \times 0.926 = 0.073 \text{ or } 7.3 \text{ per cent.} \end{aligned}$$

(h) Heat units per i.h.p. per minute.

We know that one i.h.p. represents 33,000 foot-pounds of work, and furthermore that one B.t.u. equals 778 foot-pounds. From this relation we find that the number of B.t.u. per i.h.p. per minute (no losses) is $\frac{33,000}{778}$, or 42.42.

But the actual thermal efficiency (E_4) is but 7.88 per cent, so the actual number of **B.t.u. necessary per minute** will equal $\frac{42.42}{0.0788}$, or 538 B.t.u.

II. Compound High-speed Non-condensing Steam Engine.

Assume that this engine works under the same pressure conditions as the above engine with the steam quality at unity. Find all efficiencies with the number of B.t.u. required per i.h.p. per minute when it operates at 130 i.h.p. and uses 25 pounds of steam per hour. Prony brake test gives 110 b.h.p.

NOTE. The values for the efficiencies, E_1 , E_2 and E_3 will be the same for this engine as for the case of the simple engine as just calculated.

- (a) Maximum Possible Efficiency (E_1) = 16.14 per cent.
- (b) Theoretical Thermal Efficiency (E_2) = 11.5 per cent.
- (c) Type Efficiency (E_3) = 71.3 per cent.
- (d) Actual Thermal Efficiency (E_4).

$$\begin{aligned} E_4 &= \frac{2545^*}{25(1189 - 181)} \\ &= \frac{2545}{25 \times 1008} = 0.101 \text{ or } 10.1 \text{ per cent.} \end{aligned}$$

* See page 146 (foot-note).

(e) Mechanical Efficiency (E_6).

$$\begin{aligned} E_6 &= \frac{\text{b.h.p.}}{\text{i.h.p.}} \\ &= \frac{11.0}{13.0} = 0.847 \text{ or } 84.7 \text{ per cent.} \end{aligned}$$

(f) Practical Efficiency (E_5).

$$\begin{aligned} E_5 &= \frac{E_4}{E_2} \\ &= \frac{10.1}{11.5} = 0.878 \text{ or } 87.8 \text{ per cent.} \end{aligned}$$

(g) Overall Efficiency (E_7).

$$\begin{aligned} E_7 &= E_4 \times E_6 \\ &= 0.101 \times 0.847 = 0.0854 \text{ or } 8.54 \text{ per cent.} \end{aligned}$$

(h) Heat per i.h.p. per minute.

Refer to the former problem for the method of obtaining the value of 42.42 B.t.u. per i.h.p. per minute (no losses).

$$\begin{aligned} \text{B.t.u. per i.h.p. per minute} &= \frac{42.42}{0.101}, \\ &= 420 \text{ B.t.u.} \end{aligned}$$

III. Steam Turbine. A turbine using steam at 200 pounds per square inch absolute pressure at 160° F. superheat and a **back pressure** of 0.5 pound per square inch absolute with a barometer of 29.92 inches (of mercury), consumes 12 pounds of steam per kilowatt-hour. Find maximum possible efficiency, the theoretical thermal efficiency, the type efficiency, actual thermal efficiency and the practical efficiency, for such conditions. Calculate the heat consumption per kilowatt-minute and also per electrical horse-power-minute.

(a) Maximum Possible Efficiency (E_1).

$$\begin{aligned} E_1 &= \frac{T_1 - T_2}{T_1}, \\ &= \frac{(541.9 + 460) - (80 + 460)}{541.9 + 460}, \\ &= \frac{461.9}{1001.9} = 0.461 \text{ or } 46.1 \text{ per cent.} \end{aligned}$$

(b) Theoretical Thermal Efficiency (E_2),

$$E_2 = \frac{1288 - 886}{1288 - 80},$$

where 1288 is the total heat in B.t.u. per pound at 200 pounds per square inch absolute, from which point the expansion takes place adiabatically down to 0.5 pound per square inch absolute. The total heat at the lower pressure is 886 B.t.u. per pound. The heat of the liquid at 0.5 pound per square inch absolute (back pressure) is 80 B.t.u. per pound, or,

$$E_2 = \frac{402}{1208} = 0.333 \text{ or } 33.3 \text{ per cent.}$$

(c) Type Efficiency, $E_3 = \frac{E_2}{E_1}$.

$$E_3 = \frac{0.333}{0.461} = 0.722 \text{ or } 72.2 \text{ per cent.}$$

(d) Actual Thermal Efficiency, $E_4 = \frac{W}{H_a}$.

$$\text{Then, } E_4 = \frac{2545^*}{12 (1288 - 80) \times 0.746}$$

NOTE. Denominator is multiplied by the value 0.746 because the number of horse power multiplied by this coefficient gives the equivalent power in kilowatts.

$$\begin{aligned} E_4 &= \frac{2545}{12 \times 1208 \times 0.746}, \\ &= 0.226 \text{ or } 22.6 \text{ per cent.} \end{aligned}$$

(e) Practical Efficiency, $E_5 = \frac{E_4}{E_2}$.

(f) Heat per kilowatt-minute,

$$E_5 = \frac{22.6}{33.3} = 0.679 \text{ or } 67.9 \text{ per cent.}$$

We know that there are 42.42 B.t.u. per horse power per minute (no losses) and that one horse power equals 0.746 kilowatt. Then, similarly **for no losses**, we have

$$\frac{42.42}{0.746} = 56.6 \text{ B.t.u. per kilowatt per minute.}$$

* See page 146 (foot-note).

But the test shows an **actual thermal efficiency** of 22.6 per cent and the actual heat unit consumption must be established on this basis, thus,

$$\text{B.t.u. per kilowatt per minute} = \frac{56.6}{0.226} = 250 \text{ B.t.u.}$$

The heat per electrical horse power per minute can readily be found by multiplying the B.t.u. per kilowatt-minute by 0.746 (which is the kilowatt equivalent of a horse power), or $250 \times 0.746 = 187$ B.t.u. per electrical horse power per minute.

IV. Unit Consisting of Steam Boiler, Steam Engine and Electric Generator. A "locomobile" engine* gives the following test results: i.h.p. = 185; kilowatts of generator = 115; steam consumption per i.h.p. per hour = 10 pounds; steam pressure = 210.3 pounds per square inch gage; superheat = 260° F.; barometer, 14.7 pounds per square inch; vacuum = 12.7 pounds per square inch (nearly 26 inches of mercury); feed water temperature = 142° F.

Find the various efficiencies, E_1, E_2, E_3, E_4, E_5 and E_6 and determine the number of B.t.u. per i.h.p. per minute. The coal that was used in the boiler was found to have 14,350 B.t.u. per pound. With an evaporating capacity of 9 pounds of steam per pound of coal, what is also the boiler efficiency?

(a) Maximum Possible Efficiency (E_1),

$$E_1 = \frac{T_1 - T_2}{T_1} = \frac{(T_1 + t_1) - T_2}{T_1 + t_1},$$

where t_1 is the degrees Fahrenheit of superheat; † other values are the same as in previous problems, then,

$$E_1 = \frac{(391.9 + 260 + 460) - (126 + 460)}{391.9 + 260 + 460},$$

$$\begin{aligned} E_1 &= \frac{1111.9 - 586}{1111.9} \\ &= \frac{525.9}{1111.9} = 0.473, \text{ or } 47.3 \text{ per cent.} \end{aligned}$$

* A "locomobile" is a combination of steam boiler and engine, arranged with the engine placed on top of the boiler.

† The 126° F. corresponds to (14.7 - 12.7), or 2 lbs. per sq. in. pressure. (See Steam Tables.)

(b) Theoretical Thermal Efficiency (E_2),

$$E_2 = \frac{H_1 - H_2}{H_1},$$

$$E_2 = \frac{1325 - 1005}{1325 - 94},$$

where 1325 B.t.u. is the heat read from the "Mollier Diagram" at 225 pounds per square inch absolute and 260° F. superheat. From this point the steam expands adiabatically to the lower condition of 2 pounds per square inch absolute (14.7 - 12.7) where the total heat is 1005 B.t.u. per pound; the value 94 is the heat of the liquid at 2 pounds per square inch absolute pressure. Then,

$$E_2 = \frac{320}{1231} = 0.26, \text{ or } 26 \text{ per cent.}$$

(c) Type Efficiency (E_3).

$$E_3 = \frac{26}{47.3} = 0.55, \text{ or } 55 \text{ per cent.}$$

(d) Actual Thermal Efficiency (E_4).

$$E_4 = \frac{2545}{10 \times (1325 - 94)},$$

$$= \frac{2545}{10 \times 1231} = 0.2066, \text{ or } 20.66 \text{ per cent.}$$

(e) Practical Efficiency (E_5).

$$E_5 = \frac{20.66}{26.00} = 0.794, \text{ or } 79.4 \text{ per cent.}$$

(f) Mechanical Efficiency (E_6).

$$E_6 = \frac{\text{b.h.p.}}{\text{i.h.p.}} = \frac{\text{kilowatt output}}{\text{indicated kilowatts}};$$

where "kilowatt output" of the power developed by the generator and the "indicated kilowatts" are the number of kilowatts corresponding to i.h.p. Then,

$$E_6 = \frac{115}{185 \times 0.746} = 0.833, \text{ or } 83.3 \text{ per cent.}$$

(g) B.t.u. per i.h.p. per minute.

$$\text{B.t.u. per i.h.p. per minute} = \frac{42.42}{0.2066} = 205.$$

(h) Efficiency of the Boiler.

$$\begin{aligned} \text{Eff.} &= \frac{\text{Lbs. steam evap. per lb. coal} \times \text{B.t.u. avail. per lb. of steam.}^*}{\text{B.t.u. per lb. of coal}} \\ &= \frac{9[1325 - (142 - 32)]}{14,350} \\ &= \frac{9 \times 1215}{14,350} = 0.7625, \text{ or } 76.25 \text{ per cent.} \end{aligned}$$

V. Gas-producer and Engine. If the mechanical efficiency of a producer-gas engine is known to be 85 per cent, what is the i.h.p., provided the machine is running under Prony brake test at 658 r.p.m. with a load of 400 pounds (net) at an 8-foot radius? Suppose the producer has an efficiency of 60 per cent, find the B.t.u. per i.h.p. per minute and also the B.t.u. per b.h.p. per minute if the coal consumed per i.h.p. per hour is 1 pound. Heating value of the coal is 14,000 B.t.u. per pound.

(a) Calculation of Indicated Horse Power.

$$\begin{aligned} \text{Mechanical Efficiency } (E_6) &= \frac{\text{b.h.p.}}{\text{i.h.p.}}, \text{ or } \text{i.h.p.} = \frac{\text{b.h.p.}}{E_6}, \\ \text{b.h.p.} &= \frac{2 \times 3.141 \times r \times N \times W}{33,000}, \end{aligned}$$

which is the equation for b.h.p., where r is the radius of the brake in feet, N is the number of revolutions per minute of the engine, and W is the net load on the brake arm at the measured radius, then,

$$\text{b.h.p.} = \frac{2 \times 3.141 \times 8 \times 658 \times 400}{33,000} = 400;$$

also,

$$\text{i.h.p.} = \frac{400}{0.85} = 471.$$

* B.t.u. per pound above the temperature of feed water.

(b) B.t.u. per i.h.p. per minute.

This value is dependent on the actual thermal efficiency (E_4), thus, since

$$E_4 = \frac{2545}{0.60 \times 1.00 \times 1400},$$

$$= 0.303, \text{ or } 30.3 \text{ per cent.}$$

$$\therefore \text{ B.t.u. per i.h.p. per minute} = \frac{42.42}{0.303} = 140.$$

(c) B.t.u. per b.h.p. per minute.

Since the mechanical efficiency is 85 per cent, B.t.u. per b.h.p. per minute = $\frac{140}{0.85}$, or 164.8.

VI. Blast-furnace Gas Engine. The results of a test of a blast-furnace gas engine show that 100 cubic feet of gas is consumed per i.h.p. per hour. Heating value of gas was 120 B.t.u. per cubic foot; the i.h.p. developed 800 and b.h.p. = 580. Find the mechanical efficiency (E_6) of the machine. Find the actual thermal efficiency (E_4). What are the heat equivalents B.t.u. per i.h.p. per minute, and also per b.h.p.?

(a) Mechanical Efficiency (E_6).

$$E_6 = \frac{580}{800} = 0.725, \text{ or } 72.5 \text{ per cent.}$$

(b) Actual Thermal Efficiency (E_4).

$$E_4 = \frac{2545}{120 \times 100} = 0.212, \text{ or } 21.2 \text{ per cent.}$$

(c) B.t.u. per i.h.p. per minute.

$$\text{B.t.u. per i.h.p. per minute} = \frac{42.42}{0.212} = 200.$$

(d) B.t.u. per b.h.p. per minute.

Since the mechanical efficiency was found to be 72.5 per cent,

$$\text{B.t.u. per b.h.p. per minute} = \frac{200}{0.725}, \text{ or } 276.$$

VII. Oil Engine. In tests of an oil engine, the switchboard readings showed a generator output of 375 kilowatts, and the in-

indicator diagrams 560 i.h.p. If the machine consumed 0.32 pounds of oil per i.h.p. per hour, what are the efficiencies, E_4 and E_6 ? How many B.t.u. were required per b.h.p. and i.h.p. per minute? Heat value of the oil was 19,000 B.t.u. per pound.

(a) Mechanical Efficiency (E_6).

The output of the machine is 375 kilowatts which is equal to $\frac{375}{0.746}$, or 502 "electrical" horse power. From this, we can say,

$$E_6 = \frac{502}{560} = 0.8975, \text{ or } 89.75 \text{ per cent.}$$

(b) Actual Thermal Efficiency (E_4).

$$E_4 = \frac{2545}{0.32 \times 19,000} = 0.418, \text{ or } 41.8 \text{ per cent.}$$

(c) B.t.u. required per i.h.p. per minute.

$$\text{B.t.u. per i.h.p. per minute} = \frac{42.42}{0.418} = 101.4.$$

(d) B.t.u. required per b.h.p. per minute.

$$\text{B.t.u. per b.h.p. per minute} = \frac{101.4}{0.8975} = 113.1.$$

VIII. Pumping Engine. A pumping engine uses 12 pounds of steam per i.h.p. per hour under the following conditions: steam pressure = 190 pounds per square inch absolute; quality of steam = 0.98; barometer = 14.8 pounds per square inch; pressure at end of expansion = 6 pounds per square inch; back pressure = 2.2 pounds per square inch absolute. What are the various efficiencies if the i.h.p. is 850 and the "delivery" horse power 825? Find the number of B.t.u. per i.h.p. per minute, and also the "duty" per million B.t.u.

(a) Maximum Possible Efficiency (E_1).

$$\begin{aligned} E_1 &= \frac{(377.6 + 460) - (130 + 460)}{377.6 + 460} \\ &= \frac{247.6}{837.6} = 0.2952, \text{ or } 29.52 \text{ per cent.} \end{aligned}$$

(b) Theoretical Thermal Efficiency (E_2).

Using the "Mollier Diagram,"

$$\begin{aligned} E_2 &= \frac{1181 - 946 + \frac{1}{7} \frac{4}{7} \frac{4}{8} \times (6 - 2.2) \times 0.811 \times 61.89}{1181 - 97} \\ &= \frac{1181 - 946 + 35.3}{1181 - 97} \\ &= \frac{270.3}{1084} = 0.249, \text{ or } 24.9 \text{ per cent.} \end{aligned}$$

(c) Mechanical Efficiency (E_6).

$$E_6 = \frac{825}{850} = 971, \text{ or } 97.1 \text{ per cent.}$$

(d) Type Efficiency (E_3).

$$E_3 = \frac{21.2}{24.9} = 0.851, \text{ or } 85.1 \text{ per cent.}$$

(e) Actual Thermal Efficiency (E_4).

$$\begin{aligned} E_4 &= \frac{2545}{12 \times (1181 - 97)} \\ &= 0.1959, \text{ or } 19.59 \text{ per cent.} \end{aligned}$$

(f) Practical Efficiency (E_5).

$$E_5 = \frac{19.59}{24.9} = 0.786, \text{ or } 78.6 \text{ per cent.}$$

(g) B.t.u. per i.h.p. per minute.

$$\text{B.t.u. per i.h.p. per minute} = \frac{42.42}{0.1959} = 216.4.$$

(h) Duty per Million B.t.u.

Pumping engines are usually reported as being capable of a certain "duty," which is the amount of useful work done in foot-pounds per million B.t.u. supplied. Duty per million B.t.u. = $778 \times 1,000,000 \times E_7$, where E_7 is the **overall efficiency**. Then,

$$\begin{aligned} E_7 &= E_4 \times E_6, \text{ and in this case} \\ &= 0.1959 \times 0.971 = 0.19, \text{ or } 19 \text{ per cent.} \end{aligned}$$

Duty of the pump per million B.t.u. = $778 \times 1,000,000 \times 0.19$, or 147,900,000 foot-pounds.

PROBLEMS

1. Steam at a pressure of 100 lbs. per sq. in. absolute having a quality of 0.50 expands adiabatically to 15 lbs. per sq. in. absolute. What is the quality at the end of the expansion? *Ans.* 0.50.

2. One pound of steam at a pressure of 100 lbs. per sq. in. absolute has a volume of 4 cu. ft. and expands adiabatically to 15 lbs. per sq. in. absolute. What is the quality at the initial and final conditions?

Ans. 0.905; 0.817.

What is the work done during the expansion? *Ans.* 114 B.t.u.

3. Two pounds of steam having a temperature of 330° F. and quality of 0.90 expand adiabatically to 230° F.

What is the quality at end of expansion? *Ans.* 0.826.

What is the work of the expansion? *Ans.* 193 B.t.u.

4. One pound of steam having a pressure of 125 lbs. per sq. in. absolute and volume of 4.17 cu. ft. expands adiabatically to 25 lbs. per sq. in. absolute.

What is the quality at the initial and final conditions?

Ans. 100 degrees Sup.; 0.953.

What is the work of expansion? *Ans.* 95.6 B.t.u.

5. Given the steam as stated in problem 4 but expansion complete at 100 lbs. per sq. in. absolute. What is the quality at 100 lbs. pressure?

Ans. 70° F. Sup.

6. What would the pressure be if the steam in problem 4 were expanded adiabatically until it became dry and saturated?

Ans. 56.6 lbs. per sq. in. absolute.

7. Two pounds of steam having a pressure of 100 lbs. per sq. in. absolute and a temperature of 377.8° F. expand adiabatically to 15 lbs. per sq. in. absolute.

What is the quality at the initial and final conditions?

Ans. 50 degrees F. Sup.; 0.917.

What is the work of the expansion? *Ans.* 241 B.t.u.

8. One pound of dry and saturated steam has a pressure of 100 lbs. per sq. in. absolute, and expands to 20 lbs. per sq. in. absolute along an $n = 1$ curve.

What is the quality at end of the expansion?

Ans. 55° F. Sup.

What is the work of the expansion? *Ans.* 102,750 ft.-lbs.

What heat is required? *Ans.* 127.6 B.t.u.

9. Two pounds of saturated steam at a temperature of 300° F. have a total volume of 12 cu. ft., and expand to a pressure of 15 lbs. per sq. in. absolute along an $n = 1$ curve.

What is the quality at the initial and final conditions?

Ans. 0.928; 12° F. Sup.

What is the work of the expansion?

Ans. 173,000 ft.-lbs.

What heat is required?

Ans. 309 B.t.u.

10. One pound of steam at a temperature of 360° F. has a quality of 0.50, and expands under constant pressure to a volume of 3.4 cu. ft.

What is the quality at the final condition?

Ans. 90 degrees F. Sup.

What is the work of the expansion?

Ans. 9750 ft.-lbs.

What heat is required?

Ans. 482.1 B.t.u.

11. Two pounds of steam at a pressure of 100 lbs. per sq. in. absolute have a volume of 4 cu. ft., and expand under constant temperature to a volume of 8 cu. ft.

What is the quality at the initial and final conditions?

Ans. 0.452; 0.904.

What is the work of the expansion?

Ans. 57,600 ft.-lbs.

How much heat is required?

Ans. 401 B.t.u.

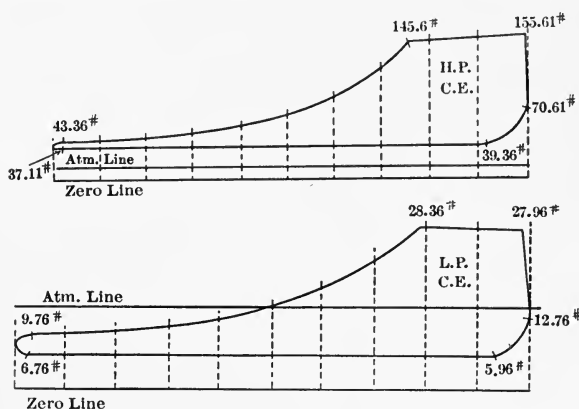


FIG. 48.—High and Low Pressure Indicator Diagram of Compound Steam Engine.

Combined Indicator Card of Compound Engine. The method of constructing indicator diagrams to a common scale of volume and pressure shows where the losses peculiar to a compound steam engine occur, and, to the same scale, the relative work areas.

As the first step divide the length of the original indicator diagrams into any number of equal parts (Fig. 48), erecting perpendiculars at the points of division.

In constructing a combined card, select a scale of absolute pressure for the ordinates and a scale of volumes in cubic feet for the abscissas. To the scale adopted draw in the atmospheric pressure ("atm.") line, see Fig. 49.

Lay off the low-pressure clearance volume on the x -axis to the scale selected. In like manner lay off the piston displacement of the low-pressure cylinder and divide this length into the same number of equal parts as the original indicator diagrams were divided. From the original low-pressure card (Fig. 48), determine the pressures at the points of intersection of the perpendiculars erected above the line of zero pressure, taking care that the proper indicator-spring scale is used. Lay off these pressures along the ordinates (Fig. 49), connect the points and the result will be the low-pressure diagram transferred to the new volume and pressure scales.

The high-pressure diagram is transferred to the new volume and pressure scale by exactly the same means as described for the low-pressure diagram.

The saturation curve is next drawn. This curve represents the curve of expansion which would be obtained if all the steam in the cylinder was dry and saturated. It is very probable that these curves for each cylinder will not be continuous since the weight of the cushion steam in the low-pressure cylinder is usually not the same as that in the high-pressure cylinder. The saturation curve would be continuous for the two cards only when the weight of cushion steam in the high-pressure and low-pressure cylinders is the same (assuming no leakage or other losses).

On the assumption that the steam caught in the clearance spaces at the beginning of compression is dry and saturated, the weight of the cushion steam can be calculated from the pressure at the beginning or end of compression, the corresponding cylinder volumes and the specific volumes corresponding to the pressure (as obtained from steam-tables).

The total weight of steam in the cylinder is the weight of steam taken into the engine per stroke plus the weight of steam caught

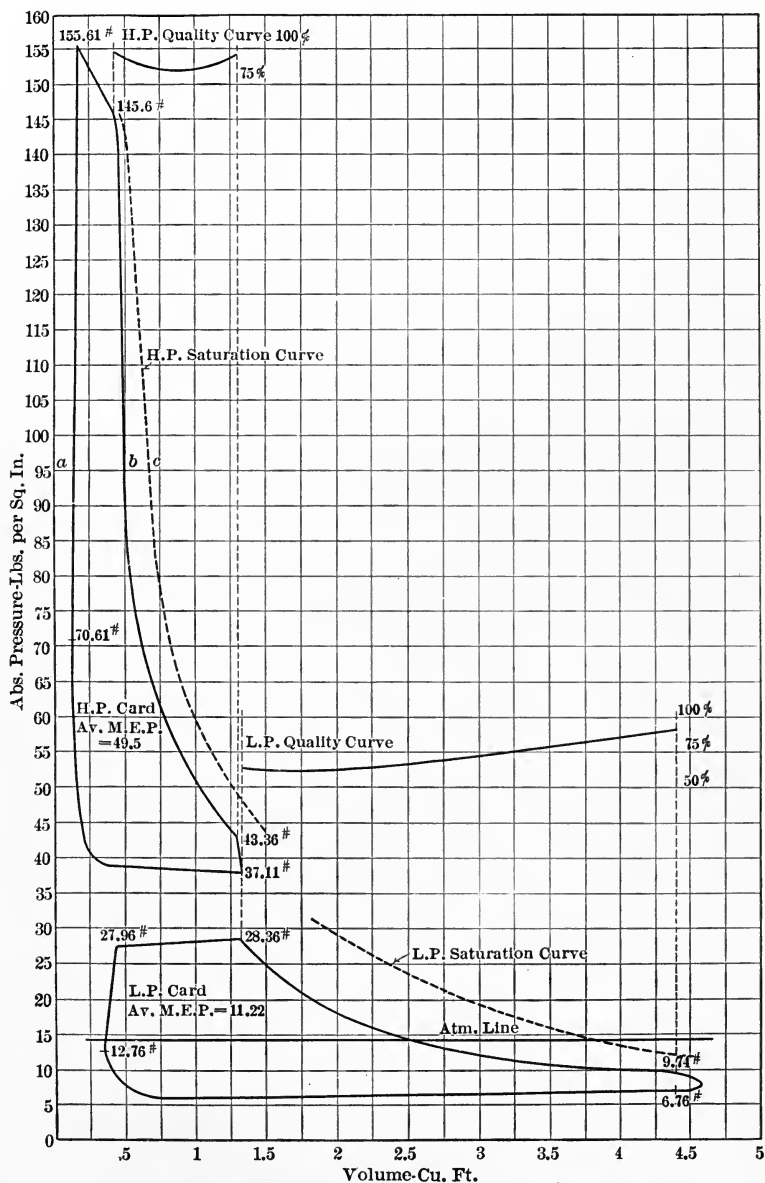


FIG. 49. — Combined Indicator Diagrams for Compound Engine.

in the clearance space (cushion steam). The saturation curves may now be drawn by plotting the volumes which the total weight of steam will occupy at different pressures, assuming it to be dry throughout the stroke.

The quality curve (Fig. 49) shows the condition of the steam as the expansion goes on. At any given absolute pressure, the volume up to the expansion line shows the volume of the wet vapor, while the volume up to the saturation curve shows the volume that the weight of the wet vapor would have if it were dry. Thus at any given absolute pressure, the ratio of the volume of the wet vapor (as given by the expansion line of the indicator card) to the total volume of the dry vapor (as obtained from the saturation curve) is the measure of the quality of the steam.

Showing the quality by the use of the figure, we have $Vol. A-B \div Vol. A-C = \text{quality}$. By laying off this ratio from a horizontal line to any scale desired as shown, the quality curve may be constructed.

Hirn's Analysis. By the use of an indicator diagram, it is possible to determine exactly the heat added to or lost from the steam. Hirn's analysis shows on a theoretical basis these gains and losses for each condition of the steam. Investigations have shown, however, that the method is of no great value except to designers.

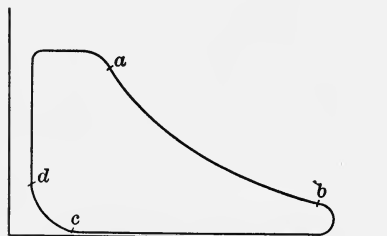


FIG. 50. — Indicator Diagram to Illustrate Hirn's Analysis.

Fig. 50 is an indicator card, compression beginning at *c*. If the quality of steam at this

point were known, the weight of and the heat in the steam can obviously be calculated.

Assuming the correct quality at *c* is known, the heat added to or lost from the steam from *c* to *d* and for each point from *c* to *d* can be exactly determined. Laying off, for each point, those quantities above the zero line (Fig. 51), the shape of the

area cdd' is found which represents the amount of heat added to the steam.

From d to a steam is taken in from the boiler. Determining this weight from the boiler or from the condenser, the total weight as well as the volume at a is known, thus the entropy at a can be calculated. By comparing the conditions of

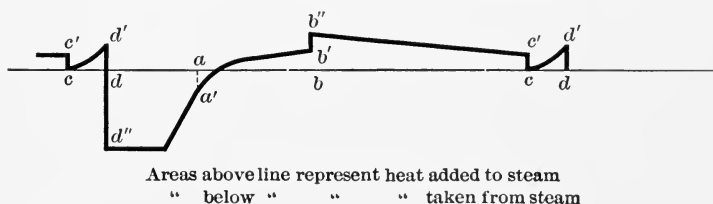


FIG. 51.—Diagram of Hirn's Analysis.

the steam when entering and at a , the loss to the steam due to the heat stored up in the cylinder walls is shown by the area $dd''a'a$. The shape of this part of the diagram is unknown although its area is known if the original assumption at c is correct. A reasonable guess of the shape of the area may be made. The greater part of the initial condensation probably takes place at the beginning of admission and falls off very fast as the point of cut-off is approached, thus dd'' would be very large and $a'a$ very small.

From a to b the cylinder walls give up heat to the steam which in turn loses its heat because of the work done. The condition of the steam at every point along ab can be determined, and thus the shape of the area is also determined.

The exhaust opens at b , the cylinder walls give up their heat to the steam at a greater rate than before because the steam is being discharged.

The area $dd''a'a$ is the quantity of heat exchanged between the cylinder walls and the steam during the time steam is taken into the cylinder and the results practically mean that during admission so much heat must be stored up somewhere. The heat interchanged during expansion is worked out on the basis that

if the weight and pressure of a given weight of steam are known at the beginning and end of a certain period, the difference in heat is accounted for by the work done and the heat added to or lost from the cylinder walls.

CHAPTER IX

FLOW OF FLUIDS

Flow through a Nozzle or Orifice. Thermodynamic problems embrace the measurement of the flow of air or of a mixture of a liquid and vapor through a nozzle or orifice. In the nozzle shown in **Fig. 52**, let **A** and **B** be two sections through which the substance passes. At **A** let a pressure of P_1 be maintained and at **B** a pressure of P_2 . To maintain the constant pressure at **A** of P_1 let more substance be added, while at **B** allow enough of the substance to be discharged so that the constant pressure of P_2 is maintained. The quantity of energy and the mass passing into the section **A** must be accounted for at section **B** and the relation of these quantities will determine the change of the velocity of the substance.

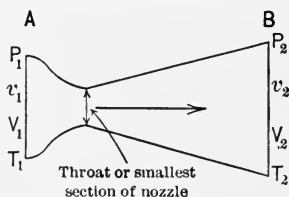


FIG. 52.—Typical Nozzle for Expanding Gases and Vapors.

After uniform conditions have been established in the nozzle, the same mass entering at **A** must be discharged at **B** during the same time. Thus any mass may be considered as a working basis, but as a rule **one pound** of the substance is used. All formulas refer to one pound, unless another mass is definitely stated.

The same quantity of energy discharged at **B** must enter at **A** unless heat is added to or taken from the substance between the sections **A** and **B**. Thus the general formula is derived:

Energy carried by substance at **B** = energy carried by substance at **A** + energy added between the sections **A** and **B**.

The energy carried by the substance at the entering or discharge end of the nozzle is made up of three quantities: (1) the amount of work necessary to maintain constant pressure at each

end of the nozzle; (2) the internal energy of the substance, (3) the kinetic energy stored in the substance because of the velocity which it has when passing the section.

The amount of work necessary to maintain a constant pressure (pounds per square foot) of P_1 at **A** or of P_2 at **B** is $\frac{P_1 v_1}{778}$ or $\frac{P_2 v_2}{778}$, where v_1 and v_2 are the volumes of one pound of the substance at **A** and **B** respectively.

The **internal** or "intrinsic" energy in **B.t.u.** per pound (I_H) of the substance passing **A** or **B**, calculating from 32°F. is for

$$\text{Air or similar gases, } \frac{R \times (T - 492)^*}{778 \times 0.40}, \quad (95)$$

$$\text{Liquid, } h, \quad (96)$$

$$\text{Liquid and Vapor, } h + x \left(L - \frac{V^2}{2g \times 778} \right), \quad (97)$$

$$\begin{aligned} \text{Superheated Vapor, } h + \left(L - \frac{V^2}{2g \times 778} \right) \\ + C_p(T_{\text{sup}} - T_{\text{sat}}) - \frac{P(v_{\text{sup}} - v_{\text{sat}})}{778}, \quad (98) \end{aligned}$$

where $\frac{V^2}{2g \times 778}$ is the kinetic energy in **B.t.u.** per pound of the substance as it passes a section, V = velocity in feet per second and $g = 32.2$ feet.

If Q represents in **B.t.u.** per pound the heat units added to the substance between the sections **A** and **B** (**Fig. 52**), the **energy equation for air** and similar gases can be found by equating the total heat energy put in at **A** plus the energy added between **A** and **B** to the energy discharged at **B**. This general formula reduces to

$$\begin{aligned} V_2^2 - V_1^2 &= 2g \left[\frac{1.4}{0.4} (P_1 v_1 - P_2 v_2) + 778 Q \right] \\ \text{or} \quad &= 2g \times 778 [C_p(T_1 - T_2) + Q]. \quad (99) \end{aligned}$$

* See equation (44), page 38.

This thermodynamic equation is the usual form for the flow of air or similar gases.

The energy equation for superheated steam can be derived by the use of the same general formula stated above, on the assumption that the substance is superheated at **A** and **B**. This formula reduces to

$$\frac{V_2^2 - V_1^2}{2g \times 778} = [h_1 + L_1 + C_p(T_{\text{sup}} - T_{\text{sat}})_1] - [h_2 + L_2 + C_p(T_{\text{sup}} - T_{\text{sat}})_2] + Q, \quad (100)$$

or
$$V_2^2 - V_1^2 = 2g \times 778 [\text{Tabular heat content}_1^* - \text{Tabular heat content}_2 + Q]. \quad (101)$$

From a thermodynamic standpoint, the relation between the initial and final condition is that of **adiabatic expansion** when all the heat which disappears as such is used in changing the velocity, providing the nozzle is properly shaped and Q is zero. The diagram in **Fig. 53** represents this condition of affairs on a temperature-entropy diagram for air and similar substances. The area **acdf** is $C_p(T_2 - 492)$, and the area **abef** is $C_p(T_1 - 492)$. The quantity of heat energy changed into kinetic energy is therefore the area **bcde** and is the difference between the internal energy in the substance at the beginning and end of the operation, together with the excess of work done to maintain the pressure of P_1 at **A** over the pressure of P_2 at **B**. The line **cd** would incline to the right if heat were added in the nozzle, since the effect would be to increase the velocity or increase the area **cdeb**. The line **cd** would incline to the left from **c** if heat were lost since the area **cdeb** would decrease.

In **Fig. 54** the diagram represents the conditions for superheated vapor. The area **aa'cdf** represents the heat required to raise the substance from a liquid at 32°F . to superheated vapor at the temperature of T_{sup} . The area **ab'bdf** is the heat content at **b**, the final condition. The area **a'cbb'** represents, there-

* Tabular heat content means the total heat of superheated steam as read from tables of the properties of superheated steam as, for example, Marks and Davis' *Tables*, pages 22-65.

fore, the heat available for increasing the velocity. The areas representing the heat available for increasing the velocity in Fig.

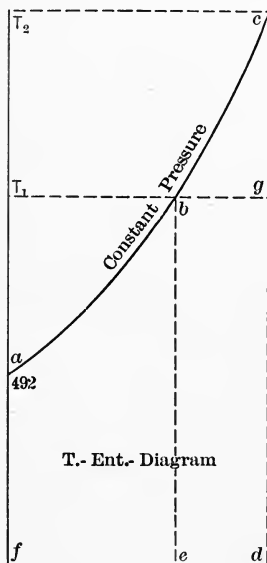


FIG. 53.—Temperature-entropy Diagram of Heat Available in Air.

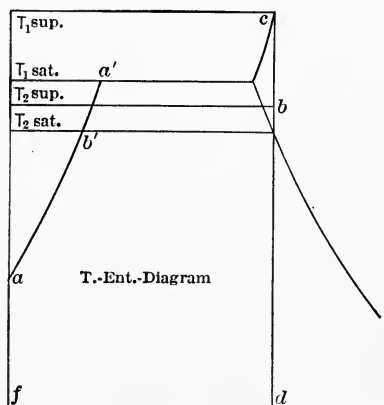


FIG. 54.—Temperature-entropy Diagram of the Heat Available in Superheated Steam for Increasing Velocity.

53 and Fig. 54 are shown by the cross-hatched area in Fig. 55 and are really the representation of the work done (theoretically) in an engine giving such an indicator diagram.

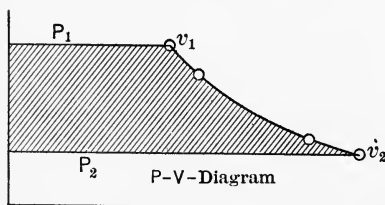


FIG. 55.—Heat (Work) Available for Increasing Velocity.

Evidently, the greater the drop in pressure, the greater will be the cross-hatched area in Fig. 55 and the greater will be the velocity, regardless of the substance. The line *ab* in Fig. 56 represents the velocity curve with *V* as ordinates and $\frac{P_2}{P_1}$ as abscissas, but since with

any substance expanding the weight of a cubic foot decreases as the pressure drops, the line *cd* will represent to some scale not here determined the weight of a cubic foot at any discharge pressure.

Since the product of the area (square feet) through which the discharge takes place, the velocity in feet per second at the area and the weight (pounds) per cubic foot of the substance is equiv-

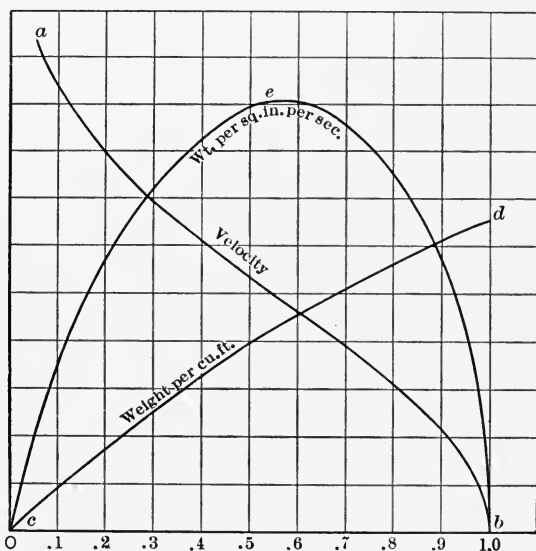


FIG. 56. — Illustrative Curves of Weight Discharge and Velocity.

alent to the weight in pounds of the substance discharged per second. The product of the ordinates at any point of the curves *ab* and *cd* is proportional to the weight discharged from a pressure of P_1 to a condition where the pressure is P_2 . The line *ceb* represents this product. Evidently there is some low pressure into which the weight discharged per square foot will be a maximum, and this will be the pressure corresponding to the high point *e* on the curve.

Weight per Cubic Foot. From the formula for adiabatic expansion the weight per cubic foot can be obtained if the sub-

stance is similar to air.* The general formula (applied to air) is

$$P_1 v_1^{1.40} = P_2 v_2^{1.40}, \quad (102)$$

which can be reduced to

$$\frac{1}{v_2} = \frac{P_2^{\frac{1}{1.4}} \times P_1}{P_1^{\frac{1}{1.4}} \times RT_1}, \quad \text{or} \quad \frac{1}{v_2} = \frac{P_1^{0.286} P_2^{0.714}}{RT_1}, \quad (103)$$

which is the weight in pounds per cubic foot of discharge. Pressures are, of course, in pounds per square foot. If the supply to the nozzle is from a large reservoir so that V_1 can be taken as zero then the discharge velocity is

$$\begin{aligned} V_2 &= \sqrt{2g \times \frac{1.4}{0.4} (P_1 v_1 - P_2 v_2)} \\ V_2 &= \sqrt{2g \times \frac{1.4}{0.4} RT_1 \left(1 - \frac{T_2}{T_1}\right)} \\ V_2 &= 109.6 \sqrt{T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{0.286}\right]}. \end{aligned} \quad (104)$$

All quantities on the right-hand side of this equation must be obtained from the data of tests. Weight in pounds discharged through the area A (in square feet) is

$$W = A \times \frac{P_1^{0.286} P_2^{0.714}}{RT_1} \times 109.6 \sqrt{T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{0.286}\right]}. \quad (105)$$

Maximum Discharge. This weight is a maximum when $\frac{dW}{dP_2} = 0$ or when $P_2 = 0.525 P_1$. The maximum quantity of air will be discharged when the low pressure is 52.5 per cent of the high pressure.

Shape of Nozzle. See page 183, on the flow of steam.

Flow of Air through an Orifice. Air under comparatively high

* The exponent in the formula is the ratio of the specific heats (of air in this case).

pressure is usually measured in practice by means of pressure and temperature observations made on the two sides of a sharp-edged orifice in a diaphragm. The method requires the use of two pressure gages on opposite sides of the orifice and a thermometer for obtaining the temperature t_1 at the initial or higher pressure p_1 . The flow of air w , in pounds per second, may then be calculated by **Fliegner's formulas**:

$$w = 0.530 \times f \times a \frac{p_1}{\sqrt{T_1}} \text{ when } p_1 \text{ is greater than } 2 p_2, \quad (106)$$

$$w = 1.060 \times f \times a \sqrt{\frac{p_2 (p_1 - p_2)}{T_1}} \text{ when } p_1 \text{ is less than } 2 p_2, \quad (107)$$

where a is the area of the orifice in **square inches**, f is a coefficient, T_1 is the absolute initial temperature in degrees Fahrenheit at the absolute pressure p_1 in the "reservoir or high-pressure side" and p_2 is the absolute discharge pressure, both in pounds per **square inch**. When the discharge from the orifice is directly into the atmosphere, p_2 is obviously barometric pressure.

Westcott's and Weisbach's experiments show that the values of f are about 0.925 for equation (106) and about 0.63 for equation (107).

For small pressures it is often desirable to substitute manometers for pressure gages. One leg of a U-tube manometer can be connected to the high-pressure side of the orifice and the other leg to the low-pressure side. Many engineers insert valves or cocks between the manometer and the pipe in which the pressure is to be observed for the purpose of "dampening" oscillations. This practice is not to be recommended as there is always the possibility that the pressure is being throttled.* A better method is to use a U-tube made with a restricted area at the bend between the two legs. This will reduce oscillations and not affect the accuracy of the observations.

Discharge from compressors and the air supply for gas engines are frequently obtained by orifice methods.

* Report of Power Test Committee, *Journal A.S.M.E.*, Nov., 1912, page 1695.

When $p_1 - p_2$ is small compared with p_1 , the simple law of discharge* of fluids can be used as follows:

$$w = \frac{fa}{144} \sqrt{2g \times 144 (p_1 - p_2) s}, \quad (108)$$

where f is a coefficient from experiments, g is the acceleration due to gravity (32.2), and s is the unit weight of the gas measured, in pounds per cubic foot, for the average of the initial and final conditions of temperature and pressure. If the difference in pressure is measured in inches of water h with a manometer then

$$144 (p_1 - p_2) = \frac{62.4}{12} \times h \text{ (expressed in terms of pounds per square foot),}$$

$$w = \frac{fa}{144} \sqrt{2ghs \times \frac{62.4}{12}} \text{ (pounds per second),} \quad (109)$$

where 62.4 is the weight of a cubic foot of water (density) at usual "room" temperatures.

This equation can also be transformed so that a table of the weight of air is not needed, since by elementary thermodynamics $144 pv = 53.3 T$, where v is the volume in cubic feet of one

* If the density is fairly constant,

$$\frac{144 p_1}{s} + \frac{v_1^2}{2g} = \frac{144 p_2}{s} + \frac{v_0^2}{2g},$$

where v_1 is the velocity in feet per second in the "approach" to the orifice, and v_0 is the velocity in the orifice itself. Since v_1 should be very small compared with v_0 ,

$$\frac{v_0^2}{2g} = \frac{144 (p_1 - p_2)}{s},$$

$$v_0 = \sqrt{\frac{2g \times 144 (p_1 - p_2)}{s}}.$$

$$w = fa v_0 s = fas \sqrt{\frac{2g \times 144 (p_1 - p_2)}{s}},$$

or

$$w = fa \sqrt{2g \times 144 (p_1 - p_2) s}.$$

Professor A. H. Westcott has computed from accurate experiments that the value of the coefficient f in these equations is approximately 0.60.

pound and T is the absolute temperature in Fahrenheit. Since v is the reciprocal of s , then

$$s = 144 p \div 53.3 T,$$

$$\text{and} \quad w = 0.209 f a \sqrt{\frac{hp}{T_1}}. \quad (110)$$

Here p and T should be the values obtained by averaging the initial and final pressures and temperatures. Great care should be exercised in obtaining correct temperatures. When equation (47) is used, for accurate work, corrections of s for humidity must be made.*

For measurements made with orifices with a well-rounded entrance and a smooth bore so that there is practically no contraction of the jet the coefficient f in equations (47) and (48) may be taken as 0.98. In the rounding portion of the entrance to such a nozzle the largest diameter must be at least twice the diameter of the smallest section. For circular orifices with sharp corners Professor Dalby † in reporting very recent experiments stated that the coefficient for his sharp-edged orifices in a thin plate of various sizes from 1 inch to 5 inches in diameter was in all cases approximately 0.60; and these data agree very well with those published by Durley.‡

When $p_2 \div p_1 = 0.99$ the values obtained with this coefficient are in error less than $\frac{1}{2}$ per cent; and when $p_2 \div p_1 = 0.93$ the error is less than 2 per cent.

Receiver Method of Measuring Air. None of the preceding methods are adaptable for measuring the volume of air at high pressures as in the case of measuring the discharge in tests of air compressors. Pumping air into a suitably strong receiver is a method often used. The compressor is made to pump

* Tables of the weight of air are given on page 181 and tables of humidity on page 368 in Moyer's *Power Plant Testing* (2d Edition).

† *Engineering* (London), Sept. 9, 1910, page 380, and Ashcroft in *Proc. Institution of Civil Engineers*, vol. 173, page 289.

‡ *Transactions American Society of Mechanical Engineers*, vol. 27 (1905), page 193.

against any desired pressure which is kept constant by a regulating valve on the discharge pipe:

P_1 and P_2 = absolute initial and final pressures for the test, pounds per square inch.

T_1 and T_2 = mean absolute initial and final temperatures, degrees Fahrenheit.

W_1 and W_2 = initial and final weight of air in the receiver, pounds.

V = volume of receiver, cubic feet.

$P_1V = WRT_1$, and $P_2V = W_2RT_2$, where R is the constant 53.3 for air, then weight of air pumped

$$W_2 - W_1 = \frac{V}{53.3} \left(\frac{P_2}{T_2} - \frac{P_1}{T_1} \right). \quad (III)$$

In accurate laboratory tests the humidity of the air entering the compressor should be measured in order to reduce this weight of air to the corresponding equivalent volume at atmospheric pressure and temperature.

Principal errors in this method are due to difficulty in measuring the average temperature in the receiver. Whenever practicable the final pressure should be maintained in the receiver at the end of the test until the final temperature is fairly constant.

PROBLEMS

1. Air at a temperature of 100° F. and pressure of 100 lbs. per sq. in. absolute flows through a nozzle against a back pressure of 20 lbs. per sq. in. absolute. Assuming the initial velocity to be zero, what will be the velocity of discharge? *Ans.* 1570 ft. per sec.

2. If the area at the mouth of the above nozzle is 0.0025 sq. ft. and the coefficient of discharge is unity, how many pounds of air will be discharged per minute? *Ans.* 59 pounds.

3. What will be the theoretical kinetic energy per minute of the above jet assuming no frictional losses? *Ans.* 2901 B.t.u.

Flow of Steam. In Fig. 52 suppose the sections A and B are so proportioned that the velocity of the substance passing section A is the same as that at section B. Such a condition might arise in a calorimeter or in the expansion of ammonia

through a throttling or expansion valve (**Fig. 20**), as in an ice machine. The pressure at **A** will be P_1 which is greater than the pressure at **B** of P_2 . **Fig. 57** represents the entropy diagram for such a condition. As the pressure falls from P_1 to P_2 the maximum heat available to produce

velocity through the nozzle is the area **acde**. The value of the quality, represented by the symbol x for the substance after leaving the nozzle corresponds to that of point **c** and the area **acde** is the excess kinetic energy represented by the increased velocity. This excess kinetic energy is destroyed by coming into contact with the more slowly moving particles at **B** and with the sides of the vessel. The area **acde** is equal to $(h_a + x_a L_a) - (h_c + x_c L_c)$

and the relation of x_a to x_c is obviously **adiabatic**. The area **ohdbf** equals area **oh eag**; thus the heat content at **b** is the same as at **a**. The location of **b** can be found as follows:

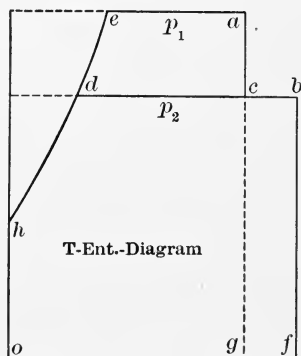


FIG. 57.—Diagram for no Velocity Change.

$$x_b = \frac{h_a + x_a L_a - h_b}{L_b}. \quad (112)$$

The curve shown in **Fig. 58** represents the discharge of a mixture of steam and water ($x = 0.6$ at 100 pounds per square inch absolute pressure) into a vessel having the pressures shown. The points on this curve cannot be determined by entropy tables. At 100 pounds per square inch pressure the total heat of the wet steam is $h + 0.6 L$ or $298.5 + 0.6 \times 887.6 = 831.1$ B.t.u. per pound.

At 60 pounds per square inch absolute pressure the total heat may be found by the use of the entropy diagram shown in **Fig. 59**. The entropy values are taken directly from steam-tables. The entropy for the initial point is, then,

$$ab = 0.4748 + 0.6 \times 1.1273 = 1.1512.$$

The distance

$$de = ab - 0.4279 = 0.7233,$$

and x at 60 pounds

$$= \frac{de}{1.2154} = \frac{0.7233}{1.2154} = 0.595.$$

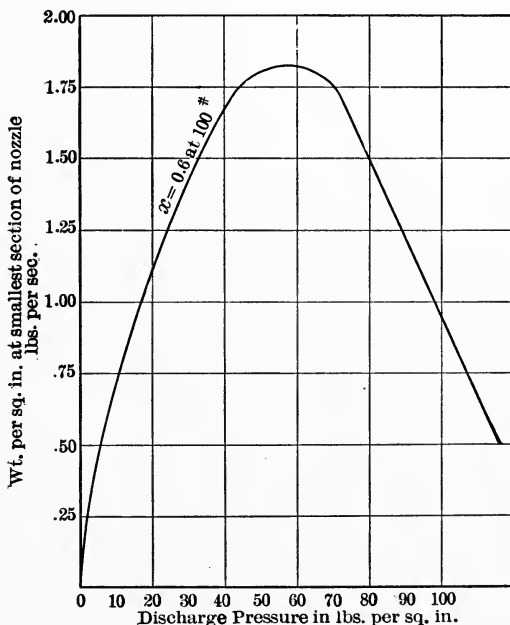


FIG. 58. — Discharge of Steam Under Various Pressures.

The total heat at 60 pounds pressure is

$$h + 0.595 L = 262.4 + 0.595 \times 914.3 = 805.4.$$

The velocity of flow is

$$\sqrt{2 \times 32.2 \times 778 (831.1 - 805.4)} = 1135 \text{ feet per second.}$$

The volume of one pound is

$$0.016 (1.0 - 0.595) + 7.166 \times 0.595 = 4.27 \text{ cubic feet,}$$

and the weight per cubic foot is

$$\frac{1}{4.27} = 0.2342 \text{ pound.}$$

The weight discharged per square inch per second is

$$\frac{1135 \times 0.2342}{144} = 1.85 \text{ pounds.}$$

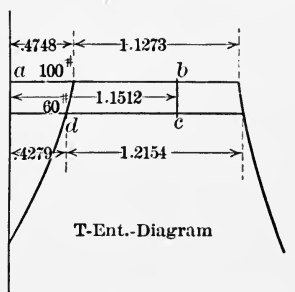


FIG. 59. — Temperature-entropy Diagram for Calculating the Weight of Discharge of Steam.

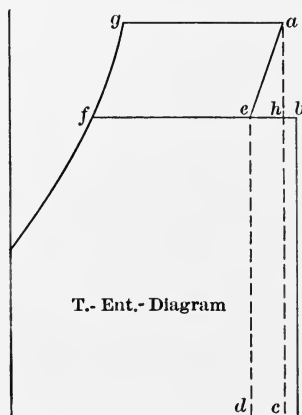


FIG. 60. — Diagram Illustrating Radiation Loss in Nozzle.

Velocity of Flow as Affected by Radiation. Fig. 60 shows the radiation losses. The condition at entrance is represented at *a* and the area *acde* represents the quantity of heat lost by radiation. Area *aefg* represents the velocity change while the point *e* represents the condition of the moving substance.

If, after passing through the nozzle, the velocity is reduced to that of entrance, a point located as at *b* will represent the condition of the substance. This point would be so located that

$$eb = \frac{\text{area } aefg}{L_b}. \quad (113)$$

Friction Loss in a Nozzle. Fig. 61 shows the friction loss. The energy converted into heat by friction varies with the square of the velocity. In this figure, *a* is the initial condition and *acfg* is the energy available for change in velocity providing there is no friction loss. The ratio of the areas *acde* to *acfg* is the proportional loss by friction. The point *c* represents the

condition of the substance at the return of the friction heat to the substance. The heat is returned in exactly the same way as if it came from an outside source. The distance **ch** is the area $\text{acde} \div L_c$. The area **edfg** represents the energy expended in the velocity change and the point **h** represents the state of the substance at the point of discharge.

This condition is found existing in the fixed nozzle of most turbines. Point **a** represents the condition on the high-pressure side of the nozzle and point **h**, the low-pressure side. The absolute velocity of discharge is really caused by the energy represented by the area **edfg**.

Impulse Nozzles. Suppose that the substance is discharged with an absolute velocity corresponding to the area **edfg** (Fig.

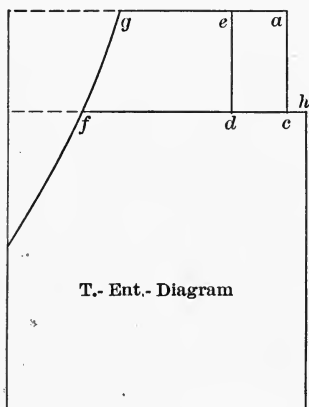


FIG. 61. — Diagram Illustrating Friction Loss in Nozzle.

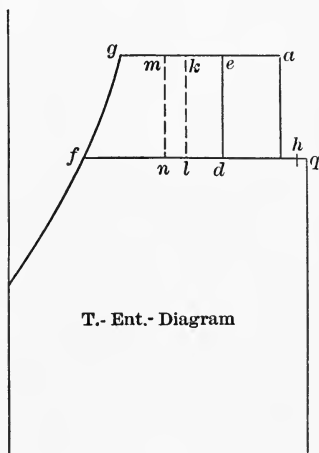


FIG. 62. — Diagram of Heat Losses in a Steam Nozzle (Impulse).

61), and that it passes into a moving nozzle, having the same pressure on the discharge as on the intake side. The energy represented by the area **edfg** would be used up in the following ways: (1) by the friction in moving nozzle; (2) residual absolute velocity; (3) and in driving the moving nozzle against the resistance.

Fig. 62 shows the quantities used up by each. Point **a** rep-

resents the condition of the substance before passing into the fixed nozzle while point **h** shows its condition leaving the fixed nozzle, the velocity corresponding to the area **edfg**. Area **klde** represents the energy used up in friction in the moving nozzle; area **klnm** residual velocity after leaving moving nozzle and area **mnfg** represents useful work used in moving the nozzle against its resistance. The condition of the substance leaving the nozzle is shown at **q** and not at **h**, the distance **h-q** being the area **edlk** divided by L_h . The substance leaves the moving nozzle with a velocity corresponding to the area **klnm** and it will have done work corresponding to the area **mnfg**.

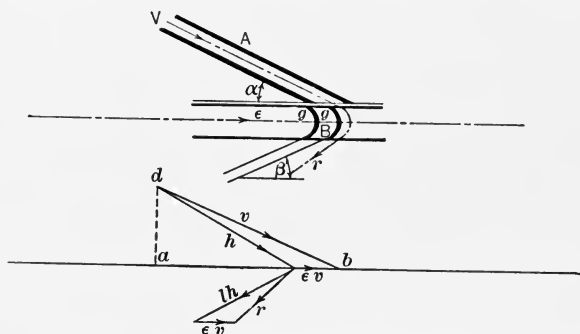


FIG. 63.—Impulse Nozzle and Velocity Diagrams.

Turbine Losses. Fig. 63 is a simple velocity diagram showing, for an impulse nozzle such as occurs in many turbines, the relative value of those various losses. **A** is a stationary nozzle discharging against the movable blades **B**. The path of the steam is shown by the dotted line. The line **db** marked *v* represents the velocity of discharge of the stationary nozzle, which makes an angle α with the direction of motion of the moving blades. Call ϵv the velocity of the moving blades, then *h* is the amount and direction of the relative velocity of the steam over the surface of the moving blades. It loses a portion of this velocity as it passes over the surface of the blades and *lh* becomes the actual relative velocity of discharge. The direction of *lh* is determined by the discharge edge of the moving blades, the angles

α and β being as shown. The residual absolute velocity is represented by r .

The total energy equivalent of the velocity developed

$$\text{in B.t.u. per pound} = \frac{v^2}{2g}. \quad (114)$$

$$\text{The residual energy per pound} = \frac{r^2}{2g}. \quad (115)$$

Reaction Nozzles. When the substance leaving the stationary nozzle passes into a moving nozzle having the pressure at the intake greater than at the discharge, the conditions differ from those just discussed. The velocity in this case is thermodynamically changed in passing through the moving nozzle. In the equations given in the previous discussion it was assumed that the moving nozzles were entirely filled with the substance,

and when partly filled in the expanding portion, coefficients of correction were applied, but in this case the nozzles should be so designed that the substance entirely fills them, as the corrections are unknown.

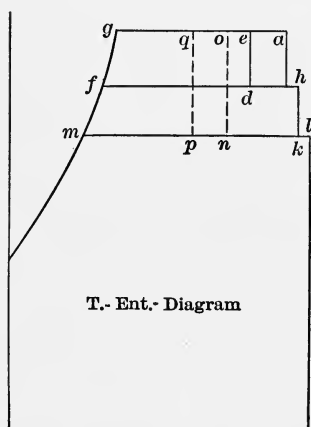


FIG. 64. — Diagram of Heat Losses in a Steam Nozzle (Reaction).

In Fig. 64 the lines of Fig. 62 are reproduced together with those relating directly to the reaction nozzle. Point a corresponds to the condition on entering the stationary nozzle, point h the condition on leaving it with a velocity corresponding to the area $edfg$. Point k represents the pressure at the discharge end of the

moving nozzle, and if no friction losses or impact loss occur in moving nozzle, point k would represent the condition of the discharged substance and the area $egmkhd$ would be accounted for as useful work done and residual velocity. But since friction losses and impact loss do occur a portion of this area $edhkno$ can be set aside to represent these losses, a portion $noqp$ represents

the residual velocity, while the remaining area **pqgm** represents the useful work done.

The condition of the substance leaving the moving nozzle is given by **k**, the distance **kl** being $\frac{\text{area edhkno}}{L_k}$. Area **onpq** represents the residual velocity.

Coefficient of Flow. Few experiments have been carried on for determining the flow of steam in nozzles proportional for maximum discharge or through nozzles exactly designed for an exact pressure. For nozzles having well rounded entrance and parallel portion of least diameter from 0.25 to 1.5 times the length of the converging entrance the coefficient of discharge is about 1.05. For properly shaped entrances and for areas of orifices between 0.125 square inch and 0.75 square inch the coefficient of discharge varies from 0.94, the two pressures being nearly alike, to unity, the ratio of the pressures being 0.57. For an orifice through a thin plate the coefficient is about 0.82, the ratio of the pressures being 0.57.

Injectors. In an injector, steam enters at *A* in Fig. 65 at the pressure of the supply. The quantity of water entering at *C*, the cross-section of the pipe, and the pressure of the water determine the pressure at *B*. At *D* the pressure should be zero (atmospheric) or equal to the pressure in the water

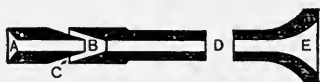


FIG. 65.—Essential Parts of an Injector.

supply pipe to which *D* may be connected. The total hydraulic head should exist as velocity head at this point. At *E* the pressure should be sufficient to raise the check valve into the boiler and the velocity sufficient to carry the intended supply into the discharge pipe.

The shape of the nozzle from *A* to *B* should be such as to convert the energy in the steam at *A* into velocity at *B*. At *B* the water and steam meet, condensing the steam, heating the water and giving to the water a velocity sufficient to carry it through the nozzle *B-D*.

All the energy accounted for at *A* and *C* must be accounted for

at E . The heat lost by radiation may be neglected. The velocity at any section of the nozzle equals

$$\frac{\text{volume passing in cubic feet}}{\text{area of section in square feet}},$$

or $V_A = \frac{aV}{a_A}$, where a = area at any section corresponding to velocity V and a_A = area at section A .

Weight of Feed Water per Pound of Steam. Assuming the steam supply to be dry and reckoning from 32° F. the heat units contained in the steam and feed water per pound and the heat in the mixture of steam and feed water per pound may be easily calculated.

Knowing the rise of temperature of the water passing through the injector and neglecting radiation losses, the pounds of feed water supplied per pound of steam used by the injector may be obtained. Thus

Heat units lost by steam = Kinetic energy of jet + Heat units gained by feed water.

The term expression "kinetic energy of jet" may be neglected since it is very small, then,

$$H - h_f = W(h_m - h_f)$$

$$W = \frac{H - h_f}{h_m - h_f}, \quad (116)$$

where W = the weight of feed water lifted per pound of steam.

h_m = heat of liquid of mixture of condensed steam and feed water.

h_f = heat of liquid of entering feed water.

Thermal Efficiency of Injector. The thermal efficiency of an injector neglecting radiation losses is unity. All the heat expended is restored either as work done or in heat returned to the boiler.

Mechanical Efficiency. The mechanical work performed by the injector consists in lifting the weight of feed water and deliver-

ing it into the boiler against the internal pressure. The efficiency, considering the injector as a pump, is

$$\frac{\text{Work done}}{\text{B.t.u. given up by steam to perform the work}},$$

or

$$E = \frac{U}{H - h_f},$$

where $U = \{ Wl_s + (W + 1) l_p \} \div 778$ (in heat units). (117)

l_p = pressure head corresponding to boiler gage pressure, in feet.

l_s = suction head in feet.

W = pounds of water delivered per pound of steam.

Orifice Measurements of the flow of steam are particularly recommended by some engineers for ascertaining the steam consumption of the "auxiliaries" in a power plant. This method commends itself particularly because of its simplicity and accuracy. It is best applied by inserting a plate $\frac{1}{8}$ inch thick with an orifice one inch in diameter, with **square** edges, at its center, between the two halves of a pair of flanges on the pipe through which the steam passes. Accurately calibrated steam gages are required on each side of the orifice to determine the loss of pressure. The weight of steam for the various differences of pressure may be determined by arranging the apparatus so that the steam passing through the orifice will be discharged into a tank of water placed on a platform scales. The flow through this orifice in pounds of dry saturated steam per hour when the discharge pressure at the orifice is 100 pounds by the gage is given by the following table:

Pressure drop, lbs. per sq. in.	Flow of dry steam per hour, lbs.	Pressure drop, lbs. per sq. in.	Flow of dry steam per hour, lbs.
$\frac{1}{2}$	430	5	1560
1	615	10	2180
2	930	15	2640
3	1200	20	3050
4	1400

Flow of Steam through Nozzles. The weight of steam discharged through any well-designed nozzle with a rounded inlet, similar to those illustrated in Figs. 66 and 67, depends only on the initial absolute pressure (P_1), if the pressure against which the nozzle discharges (P_2) does not exceed 0.58 of the initial pressure. This important statement is well illustrated by the following example. If steam at an initial pressure (P_1) of 100 pounds per square inch absolute is discharged from a nozzle, the weight of steam flowing in a given time is practically the same for all values of the pressure against which the steam is discharged (P_2) which are equal to or less than 58 pounds per square inch absolute.

If, however, the final pressure is more than 0.58 of the initial, the weight of steam discharged will be less, nearly in proportion as the difference between the initial and final pressures is reduced.

The most satisfactory and accurate formula for the "constant flow" condition, meaning when the final pressure is 0.58 of the initial pressure or less, is the following, due to Grashof,* where w is the flow of steam † (initially dry saturated) in pounds per second, A_0 is the area of the smallest section of the nozzle in

* Grashof, *Theoretische Maschinenlehre*, vol. 1, iii; *Hütte Taschenbuch*, vol. 1, page 333. Grashof states the formula,

$$w = 0.01654 A_0 P_1^{.966},$$

but the formula given in equation (118) is accurate enough for all practical uses.

† Napier's formula is very commonly used by engineers and is accurate enough for most calculations. It is usually stated in the form

$$w = \frac{A_0 P_1}{70},$$

where w , P_1 , and A_0 have the same significance as in Grashof's formula. The following formula is given by Rateau, who has done some very good theoretical and practical work on steam turbines, but this formula is too complicated for convenient use:

$$w = 0.001 A_0 P_1 [15.26 - 0.96 (\log P_1 + \log 0.0703)].$$

Common or base 10 logarithms are to be used in this formula.

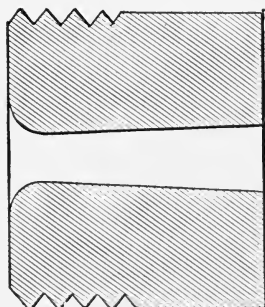
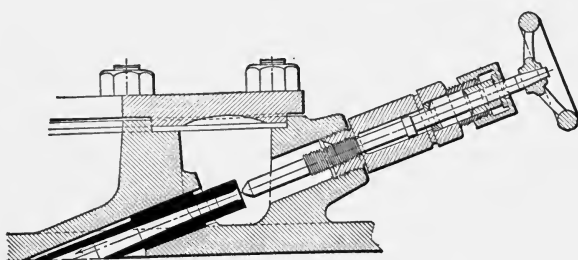
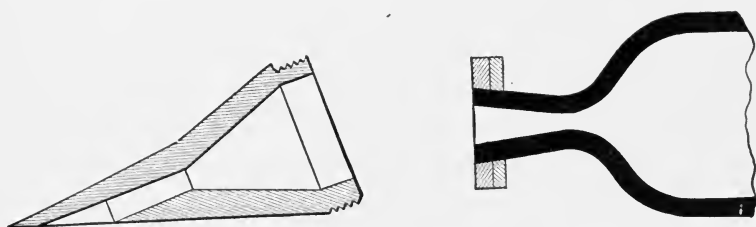


FIG. 66. — Example of a Well-designed Nozzle.



DeLaval Type.



Curtis Type.

FIG. 67. — Examples of Standard Designs of Nozzles.

square inches, and P_1 is the initial **absolute** pressure of the steam in pounds per square inch,

$$w = \frac{A_0 P_1^{.97}}{60}, \quad (118)$$

or, in terms of the area,

$$A_0 = \frac{60 w}{P_1^{.97}}. \quad (119)$$

These formulas are for the flow of steam **initially dry and saturated**. An illustration of their applications is given by the following practical example.

Example. The area of the smallest section (A_0) of a suitably designed nozzle is 0.54 square inch. What is the weight of the flow (w) of dry saturated steam per second from this nozzle when the initial pressure (P_1) is 135 pounds per square inch absolute and the discharge pressure (P_2) is 15 pounds per square inch absolute?

Here P_2 is less than 0.58 P_1 and Grashof's formula is applicable,

$$\text{or,} \quad w = \frac{0.54 (135)^{.97}}{60},$$

$$w = \frac{0.54 \times 116.5^*}{60} = 1.049 \text{ pounds per second.}$$

When steam passes through a series of nozzles one after the other as is the case in many types of turbines, the pressure is reduced and the steam is condensed in each nozzle so that it becomes wetter and wetter each time. In the low-pressure nozzles of a turbine, therefore, the steam may be very wet although initially it was dry. Turbines are also sometimes designed to operate with **steam which is initially wet**, and this is usually the case when **low-pressure steam turbines** are operated with the exhaust from non-condensing reciprocating engines — a practice which is daily becoming more common. In all these cases the nozzle area must be corrected for the wetness of the steam.

* The flow (w) calculated by Napier's formula for this example is $w = \frac{0.54 \times 135}{70}$
1.041 pounds per second.

For a given nozzle the weight discharged is, of course, greater for wet steam than for dry; but the percentage increase in the discharge is not nearly in proportion to the percentage of moisture as is often stated. The general equation for the theoretic discharge (w) from a nozzle is in the form *

$$w = K \sqrt{\frac{P_1}{v_1}},$$

where P_1 is the initial absolute pressure and v_1 is the specific volume (cubic feet in a pound of steam at the pressure P_1). Now, neglecting the volume of the water in wet steam, which is a usual approximation, the volume of a pound of steam is proportional

* The general equation for the theoretic flow is

$$w = A_0 \sqrt{\frac{2 g k P_1}{(k-1) v_1} \left[\left(\frac{P_2}{P_1} \right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1} \right)^{\frac{k+1}{k}} \right]},$$

where the symbols w , A_0 , P_1 , and g are used as in equations (117) and (118). P_2 is the pressure at any section of the nozzle, v_1 is the volume of a pound of steam at the pressure P_1 , and k is a constant. The flow, w , has its maximum value when

$$\left(\frac{P_2}{P_1} \right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1} \right)^{\frac{k+1}{k}}$$

is a maximum. Differentiating and equating the first differential to zero gives

$$\frac{P_2}{P_1} = \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}}.$$

P_2 is now the pressure at the *smallest section*, and writing for clearness P_0 for P_2 , and substituting this last equation in the formula for flow (w) above, we have

$$w = A_0 \sqrt{\frac{2 g k}{k+1} \left(\frac{P_0}{P_1} \right)^{\frac{2}{k}} \left(\frac{P_1}{v_1} \right)}.$$

Now regardless of what the final pressure may be, the pressure (P_0) at the smallest section of a nozzle (A_0) is always nearly 0.58 P_1 for dry saturated steam. Making then in the last equation $P_0 = 0.58 P_1$ and putting for k Zeuner's value of 1.135 for dry saturated steam, we may write in general terms the form stated above,

$$w = K \sqrt{\frac{P_1}{v_1}},$$

where K is another constant. See Zeuner's *Theorie der Turbinen*, page 268 (Ed. of 1899).

to the quality (x_1). For **wet steam** the equation above becomes then -

$$w = K \sqrt{\frac{P_1}{x_1 v_1}}.$$

The equation shows, therefore, that the **flow of wet steam is inversely proportional to the square root of the quality (x_1)**. Grashof's equations can be stated then more generally as

$$w = \frac{A_0 P_1^{.97}}{60 \sqrt{x_1}}, \quad (120)$$

$$A_0 = \frac{60 w \sqrt{x_1}}{P_1^{.97}}. \quad (121)$$

These equations become, of course, the same as (118) and (119) for the case where $x_1 = 1$.

Flow of Steam when the Final Pressure is more than 0.58 of the Initial Pressure. For this case the discharge depends upon the final pressure as well as upon the initial. No satisfactory formula can be given in simple terms, and the flow is most easily calculated with the aid of the curve in **Fig. 68** due to Rateau. This curve is used by determining first the ratio of the final to the initial pressure $\frac{P_2}{P_1}$, and reading from the curve the corresponding coefficient showing the ratio of the required discharge to that calculated for the given conditions by either of the equations (118) or (120). The coefficient from the curve times the flow calculated from equations (118) or (120) is the required result. Obviously the discharge for this condition is always **less** than the discharge when the final pressure is equal to or less than 0.58 of the initial.

Length for Nozzles. Probably the best designers make the length of the nozzle depend only on the initial pressure. In other words, the length of a nozzle for 150 pounds per square inch initial pressure is usually made the same for a given type regardless of the final pressure. And if it happens that there is crowding for space, one or more of the nozzles is sometimes made a little shorter than the others.

Designers of De Laval nozzles follow practically the same "elastic" method. The divergence of the walls of non-condensing nozzles is about 3 degrees from the axis of the nozzle, and condensing nozzles for high vacuums may have a divergence

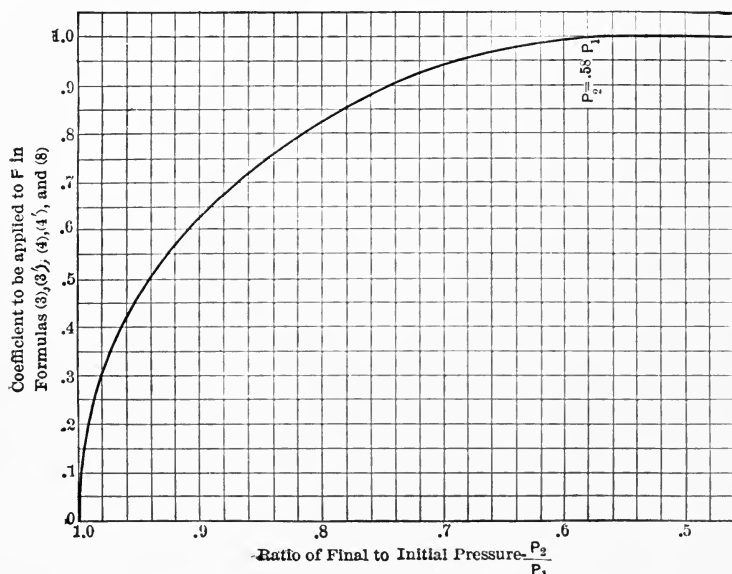


FIG. 68.—Coefficients of the Discharge of Steam when the Final-Pressure is Greater than 0.58 of the Initial Pressure.

of as much as 6 degrees * for the normal rated pressures of the turbine.

The author has used successfully the following empirical formula to determine a suitable length, L , of the nozzle between the throat and the mouth (in inches):

$$L = \sqrt{15 A_0} \dagger \quad (122)$$

where A_0 is the area at the throat in square inches.

* According to Dr. O. Recke, if the total divergence of a nozzle is more than 6 degrees, eddies will begin to form in the jet. There is no doubt that a too rapid divergence produces a velocity loss.

† Moyer's *Steam Turbines* (2d Edition), pages 45-48.

Efficiency of Nozzles. Recent experience with nozzles of this type does not bear out this statement, except in the case probably of square or rectangular nozzles with no rounding at the edges. An efficiency of 97 per cent is not unusual for properly designed square and rectangular shaped nozzles without any "square" edges; and circular nozzles have certainly never given 99 per cent efficiency.

Under- and Over-expansion. The best efficiency of a nozzle is obtained when the expansion required is that for which the nozzle was designed, or when the **expansion ratio** for the condition of the steam corresponds with the ratio of the areas of the mouth and throat of the nozzle. A little **under-expansion** is far better,

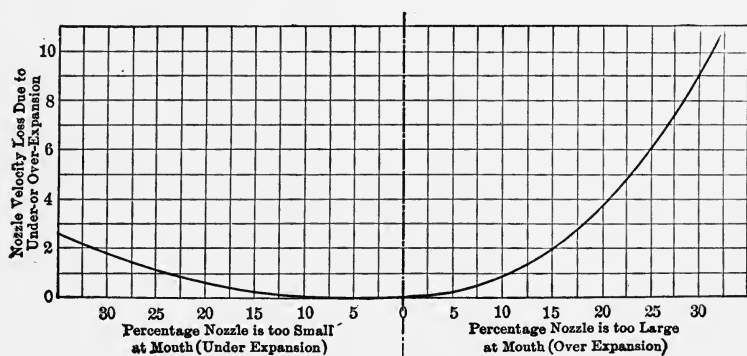


FIG. 69. — Curve of Nozzle Velocity Loss.

however, than the same amount of **over-expansion**, meaning that a nozzle that is too small for the required expansion is more efficient than one that is correspondingly too large.* Fig. 69 shows a curve representing average values of nozzle loss used by various American and European manufacturers † to determine

* It is a very good method, and one often adopted, to design nozzles so that at the rated capacity the nozzles under-expand at least 10 per cent, and maybe 20 per cent. The loss for these conditions is insignificant, and the nozzles can be run for a large overload (with increased pressures) in nearly all types without immediately reducing the efficiency very much.

† C. P. Steinmetz, *Proc. Am. Soc. Mech. Engineers*, May, 1908, page 628; J. A. Moyer, *Steam Turbines*, page 50.

discharge velocities from nozzles under the conditions of **under- or over-expansion**.

Non-expanding Nozzles. All the nozzles of Rateau turbines and usually also those of the low-pressure stages of Curtis turbines are made non-expanding; meaning, that they have the same area at the throat as at the mouth. For such conditions it has been suggested that instead of a series of separate nozzles in a row a single long nozzle might be used of which the sides were arcs of circles corresponding to the inside and outside pitch diameters of the blades. Advantages would be secured both on account of cheapness of construction and because a large amount of friction against the sides of nozzles would be eliminated by omitting a number of nozzle walls. Such a construction has not proved desirable, because by this method no well-formed jets are secured and the loss from eddies is excessive. The general statement may be made that the **throat** of a well-designed nozzle should have a nearly symmetrical shape, as for example a circle, a square, etc., rather than such shapes as ellipses and long rectangles. The shape of the mouth is not important. In Curtis turbines an approximately rectangular mouth is used because the nozzles are placed close together (usually in a nozzle plate like Fig. 67) in order to produce a continuous band of steam; and, of course, by using a section that is rectangular rather than circular or elliptical, a band of steam of more nearly uniform velocity and density is secured.

Fig. 70 shows a number of designs of non-expanding nozzles used by Professor Rateau. The length of such nozzles beyond the throat is practically negligible. Curtis non-expanding nozzles are usually made the same length as if expanding and the length is determined by the throat area.

Materials for Nozzles. Nozzles for saturated or slightly superheated steam are usually made of bronze. Gun metal, zinc alloys, and delta metal are also frequently used. All these metals have unusual resistance for erosion or corrosion from the use of wet steam. Because of this property as well as for the reason that they are easily worked with hand tools * they are

* Nozzles of irregular shapes are usually filed by hand to the exact size.

very suitable materials for the manufacture of steam turbine nozzles. Superheated steam, however, rapidly erodes all these alloys and also greatly reduces the tensile strength. For nozzles to be used with highly superheated steam, cast iron is generally used, and except that it corrodes so readily is a very satisfactory

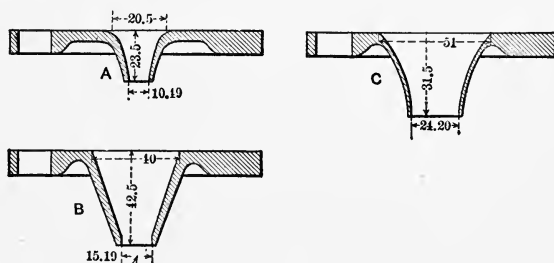


FIG. 70. — Non-expanding Nozzles.

material. Commercial copper (about 98 per cent) is said to have been used with a fair degree of success with high superheats; but for such conditions its tensile strength is very low. Steel and cupro-nickel ($8 \text{ Cu} + 2 \text{ Ni}$) are also suitable materials, and the latter has the advantage of being practically non-corrodible.

The most important part of the design of a nozzle is the determination of the areas of the various sections — especially the smallest section, if the nozzle is of an expanding or diverging type. In order to calculate the areas of nozzles we must know how to determine the quantity of steam (flow) per unit of time passing through a unit area. It is very essential that the nozzle is well rounded on the “entrance” side and that sharp edges along the path of the steam are avoided. Otherwise it is not important whether the shape of the section is circular, elliptical, or rectangular with rounded corners.

Whether the nozzle section is throughout circular, square, or rectangular (if these last sections have rounded corners) the efficiency as measured by the **velocity** will be about 96 to 97 per cent, corresponding to an equivalent **energy efficiency** of 92 to 94 per cent. Speaking commercially, therefore, it does not seem to

be worth while to spend a great deal of time in the shops to make nozzles very exactly to some difficult shape. Simpler and more rapid methods of nozzle construction should be introduced. In some shops the time of one man for two days is required for the hand labor alone on a **single** nozzle.

APPENDIX

NAPERIAN LOGARITHMS

$e = 2.7182818$

$\log e = 0.4342945 = M$

	0	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.00995	0.01980	0.02956	0.03922	0.04879	0.05827	0.06766	0.07696	0.08618
1.1	0.09531	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1739
1.2	0.1823	0.1906	0.1988	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988
1.5	0.4055	0.4121	0.4187	0.4253	0.4318	0.4382	0.4447	0.4511	0.4574	0.4637
1.6	0.4700	0.4762	0.4824	0.4886	0.4947	0.5008	0.5068	0.5128	0.5188	0.5247
1.7	0.5306	0.5365	0.5423	0.5481	0.5539	0.5596	0.5653	0.5710	0.5766	0.5822
1.8	0.5878	0.5933	0.5988	0.6043	0.6098	0.6152	0.6206	0.6259	0.6313	0.6366
1.9	0.6418	0.6471	0.6523	0.6575	0.6627	0.6678	0.6729	0.6780	0.6831	0.6881
2.0	0.6931	0.6981	0.7031	0.7080	0.7129	0.7178	0.7227	0.7275	0.7324	0.7372
2.1	0.7419	0.7467	0.7514	0.7561	0.7608	0.7655	0.7701	0.7747	0.7793	0.7839
2.2	0.7884	0.7930	0.7975	0.8020	0.8065	0.8109	0.8154	0.8198	0.8242	0.8286
2.3	0.8329	0.8372	0.8416	0.8459	0.8502	0.8544	0.8587	0.8629	0.8671	0.8713
2.4	0.8755	0.8796	0.8838	0.8879	0.8920	0.8961	0.9002	0.9042	0.9083	0.9123
2.5	0.9163	0.9203	0.9243	0.9282	0.9322	0.9361	0.9400	0.9439	0.9478	0.9517
2.6	0.9555	0.9594	0.9632	0.9670	0.9708	0.9746	0.9783	0.9821	0.9858	0.9895
2.7	0.9933	0.9969	1.0006	1.0043	1.0080	1.0116	1.0152	1.0188	1.0225	1.0260
2.8	1.0296	1.0332	1.0367	1.0403	1.0438	1.0473	1.0508	1.0543	1.0578	1.0613
2.9	1.0647	1.0682	1.0716	1.0750	1.0784	1.0818	1.0852	1.0886	1.0919	1.0953
3.0	1.0986	1.1019	1.1053	1.1086	1.1119	1.1151	1.1184	1.1217	1.1249	1.1282
3.1	1.1314	1.1346	1.1378	1.1410	1.1442	1.1474	1.1506	1.1537	1.1569	1.1600
3.2	1.1632	1.1663	1.1694	1.1725	1.1756	1.1787	1.1817	1.1848	1.1878	1.1909
3.3	1.1939	1.1969	1.2000	1.2030	1.2060	1.2090	1.2119	1.2149	1.2179	1.2208
3.4	1.2238	1.2267	1.2296	1.2326	1.2355	1.2384	1.2413	1.2442	1.2470	1.2499
3.5	1.2528	1.2556	1.2585	1.2613	1.2641	1.2669	1.2698	1.2726	1.2754	1.2782
3.6	1.2809	1.2837	1.2865	1.2892	1.2920	1.2947	1.2975	1.3002	1.3029	1.3056
3.7	1.3083	1.3110	1.3137	1.3164	1.3191	1.3218	1.3244	1.3271	1.3297	1.3324
3.8	1.3350	1.3376	1.3403	1.3429	1.3455	1.3481	1.3507	1.3533	1.3558	1.3584
3.9	1.3610	1.3635	1.3661	1.3686	1.3712	1.3737	1.3762	1.3788	1.3813	1.3838
4.0	1.3863	1.3888	1.3913	1.3938	1.3962	1.3987	1.4012	1.4036	1.4061	1.4085
4.1	1.4110	1.4134	1.4159	1.4183	1.4207	1.4231	1.4255	1.4279	1.4303	1.4327
4.2	1.4351	1.4375	1.4398	1.4422	1.4446	1.4469	1.4493	1.4516	1.4540	1.4563
4.3	1.4586	1.4609	1.4633	1.4656	1.4679	1.4702	1.4725	1.4748	1.4770	1.4793
4.4	1.4816	1.4839	1.4861	1.4884	1.4907	1.4929	1.4951	1.4974	1.4996	1.5019
4.5	1.5041	1.5063	1.5085	1.5107	1.5129	1.5151	1.5173	1.5195	1.5217	1.5239
4.6	1.5261	1.5282	1.5304	1.5326	1.5347	1.5369	1.5390	1.5412	1.5433	1.5454
4.7	1.5476	1.5497	1.5518	1.5539	1.5560	1.5581	1.5602	1.5623	1.5644	1.5665
4.8	1.5686	1.5707	1.5728	1.5748	1.5769	1.5790	1.5810	1.5831	1.5851	1.5872
4.9	1.5892	1.5913	1.5933	1.5953	1.5974	1.5994	1.6014	1.6034	1.6054	1.6074
5.0	1.6094	1.6114	1.6134	1.6154	1.6174	1.6194	1.6214	1.6233	1.6253	1.6273
5.1	1.6292	1.6312	1.6332	1.6351	1.6371	1.6390	1.6409	1.6429	1.6448	1.6467
5.2	1.6487	1.6506	1.6525	1.6544	1.6563	1.6582	1.6601	1.6620	1.6639	1.6658
5.3	1.6677	1.6696	1.6715	1.6734	1.6752	1.6771	1.6790	1.6808	1.6827	1.6845
5.4	1.6864	1.6882	1.6901	1.6919	1.6938	1.6956	1.6974	1.6993	1.7011	1.7029
5.5	1.7047	1.7066	1.7084	1.7102	1.7120	1.7138	1.7156	1.7174	1.7192	1.7210
5.6	1.7228	1.7246	1.7263	1.7281	1.7299	1.7317	1.7334	1.7352	1.7370	1.7387

NAPERIAN LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9
5.7	1.7405	1.7422	1.7440	1.7457	1.7475	1.7492	1.7509	1.7527	1.7544	1.7561
5.8	1.7579	1.7596	1.7613	1.7630	1.7647	1.7664	1.7681	1.7699	1.7716	1.7733
5.9	1.7750	1.7766	1.7783	1.7800	1.7817	1.7834	1.7851	1.7867	1.7884	1.7901
6.0	1.7918	1.7934	1.7951	1.7967	1.7984	1.8001	1.8017	1.8034	1.8050	1.8066
6.1	1.8083	1.8099	1.8116	1.8132	1.8148	1.8165	1.8181	1.8197	1.8213	1.8229
6.2	1.8245	1.8262	1.8278	1.8294	1.8310	1.8326	1.8342	1.8358	1.8374	1.8390
6.3	1.8405	1.8421	1.8437	1.8453	1.8469	1.8485	1.8500	1.8516	1.8532	1.8547
6.4	1.8563	1.8579	1.8594	1.8610	1.8625	1.8641	1.8656	1.8672	1.8687	1.8703
6.5	1.8718	1.8733	1.8749	1.8764	1.8779	1.8795	1.8810	1.8825	1.8840	1.8856
6.6	1.8871	1.8886	1.8901	1.8916	1.8931	1.8946	1.8961	1.8976	1.8991	1.9006
6.7	1.9021	1.9036	1.9051	1.9066	1.9081	1.9095	1.9110	1.9125	1.9140	1.9155
6.8	1.9169	1.9184	1.9199	1.9213	1.9228	1.9242	1.9257	1.9272	1.9286	1.9301
6.9	1.9315	1.9330	1.9344	1.9359	1.9373	1.9387	1.9402	1.9416	1.9430	1.9445
7.0	1.9459	1.9473	1.9488	1.9502	1.9516	1.9530	1.9544	1.9559	1.9573	1.9587
7.1	1.9601	1.9615	1.9629	1.9643	1.9657	1.9671	1.9685	1.9699	1.9713	1.9727
7.2	1.9741	1.9755	1.9769	1.9782	1.9796	1.9810	1.9824	1.9838	1.9851	1.9865
7.3	1.9879	1.9892	1.9906	1.9920	1.9933	1.9947	1.9961	1.9974	1.9988	2.0001
7.4	2.0015	2.0028	2.0042	2.0055	2.0069	2.0082	2.0096	2.0109	2.0122	2.0136
7.5	2.0149	2.0162	2.0176	2.0189	2.0202	2.0215	2.0229	2.0242	2.0255	2.0268
7.6	2.0281	2.0295	2.0308	2.0321	2.0334	2.0347	2.0360	2.0373	2.0386	2.0399
7.7	2.0412	2.0425	2.0438	2.0451	2.0464	2.0477	2.0490	2.0503	2.0516	2.0528
7.8	2.0541	2.0554	2.0567	2.0580	2.0592	2.0605	2.0618	2.0631	2.0643	2.0656
7.9	2.0668	2.0681	2.0694	2.0707	2.0719	2.0732	2.0744	2.0757	2.0769	2.0782
8.0	2.0794	2.0807	2.0819	2.0832	2.0844	2.0857	2.0869	2.0881	2.0894	2.0906
8.1	2.0919	2.0931	2.0943	2.0956	2.0968	2.0980	2.0992	2.1005	2.1017	2.1029
8.2	2.1041	2.1054	2.1066	2.1078	2.1090	2.1102	2.1114	2.1126	2.1138	2.1150
8.3	2.1163	2.1175	2.1187	2.1199	2.1211	2.1223	2.1235	2.1247	2.1258	2.1270
8.4	2.1282	2.1294	2.1306	2.1318	2.1330	2.1342	2.1353	2.1365	2.1377	2.1389
8.5	2.1401	2.1412	2.1424	2.1436	2.1448	2.1459	2.1471	2.1483	2.1494	2.1506
8.6	2.1518	2.1529	2.1541	2.1552	2.1564	2.1576	2.1587	2.1599	2.1610	2.1622
8.7	2.1633	2.1645	2.1656	2.1668	2.1679	2.1691	2.1702	2.1713	2.1725	2.1736
8.8	2.1748	2.1759	2.1770	2.1782	2.1793	2.1804	2.1815	2.1827	2.1838	2.1849
8.9	2.1861	2.1872	2.1883	2.1894	2.1905	2.1917	2.1928	2.1939	2.1950	2.1961
9.0	2.1972	2.1983	2.1994	2.2006	2.2017	2.2028	2.2039	2.2050	2.2061	2.2072
9.1	2.2083	2.2094	2.2105	2.2116	2.2127	2.2138	2.2148	2.2159	2.2170	2.2181
9.2	2.2192	2.2203	2.2214	2.2225	2.2235	2.2246	2.2257	2.2268	2.2279	2.2289
9.3	2.2300	2.2311	2.2322	2.2332	2.2343	2.2354	2.2364	2.2375	2.2386	2.2396
9.4	2.2407	2.2418	2.2428	2.2439	2.2450	2.2460	2.2471	2.2481	2.2492	2.2502
9.5	2.2513	2.2523	2.2534	2.2544	2.2555	2.2565	2.2576	2.2586	2.2597	2.2607
9.6	2.2618	2.2628	2.2638	2.2649	2.2659	2.2670	2.2680	2.2690	2.2701	2.2711
9.7	2.2721	2.2732	2.2742	2.2752	2.2762	2.2773	2.2783	2.2793	2.2803	2.2814
9.8	2.2824	2.2834	2.2844	2.2854	2.2865	2.2875	2.2885	2.2895	2.2905	2.2915
9.9	2.2925	2.2935	2.2946	2.2956	2.2966	2.2976	2.2986	2.2996	2.3006	2.3016
10.0	2.3026									

LOGARITHMS.

Nat. Nos.	0	1	2	3	4	5	6	7	8	9	Proportional Parts.								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS.

Nat. Nos.	0	1	2	3	4	5	6	7	8	9	Proportional Parts.								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

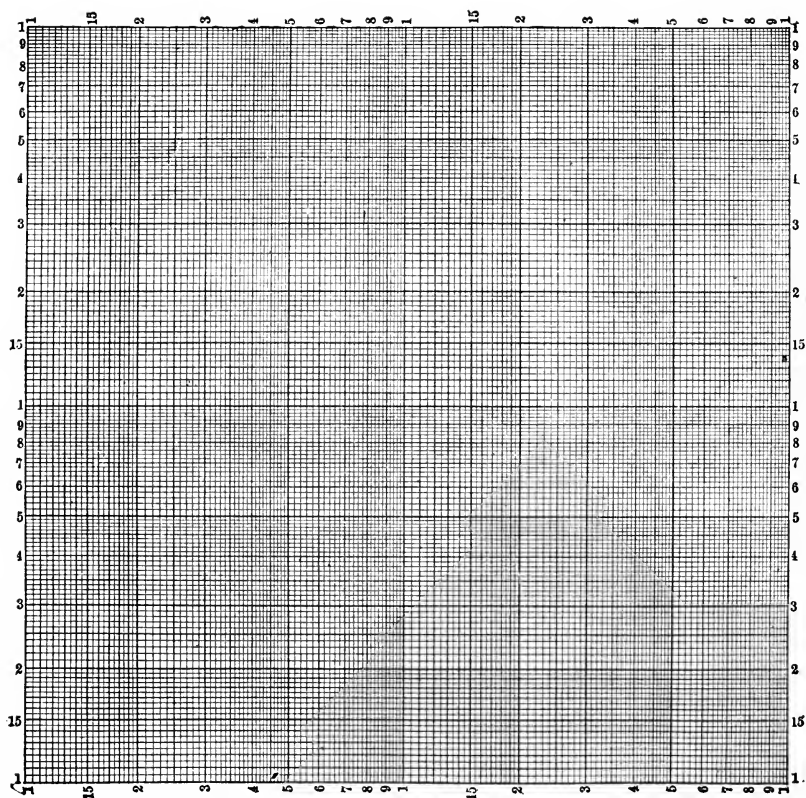


FIG. 71. — Logarithmic Cross-section Paper for Plotting Exponential Equations.

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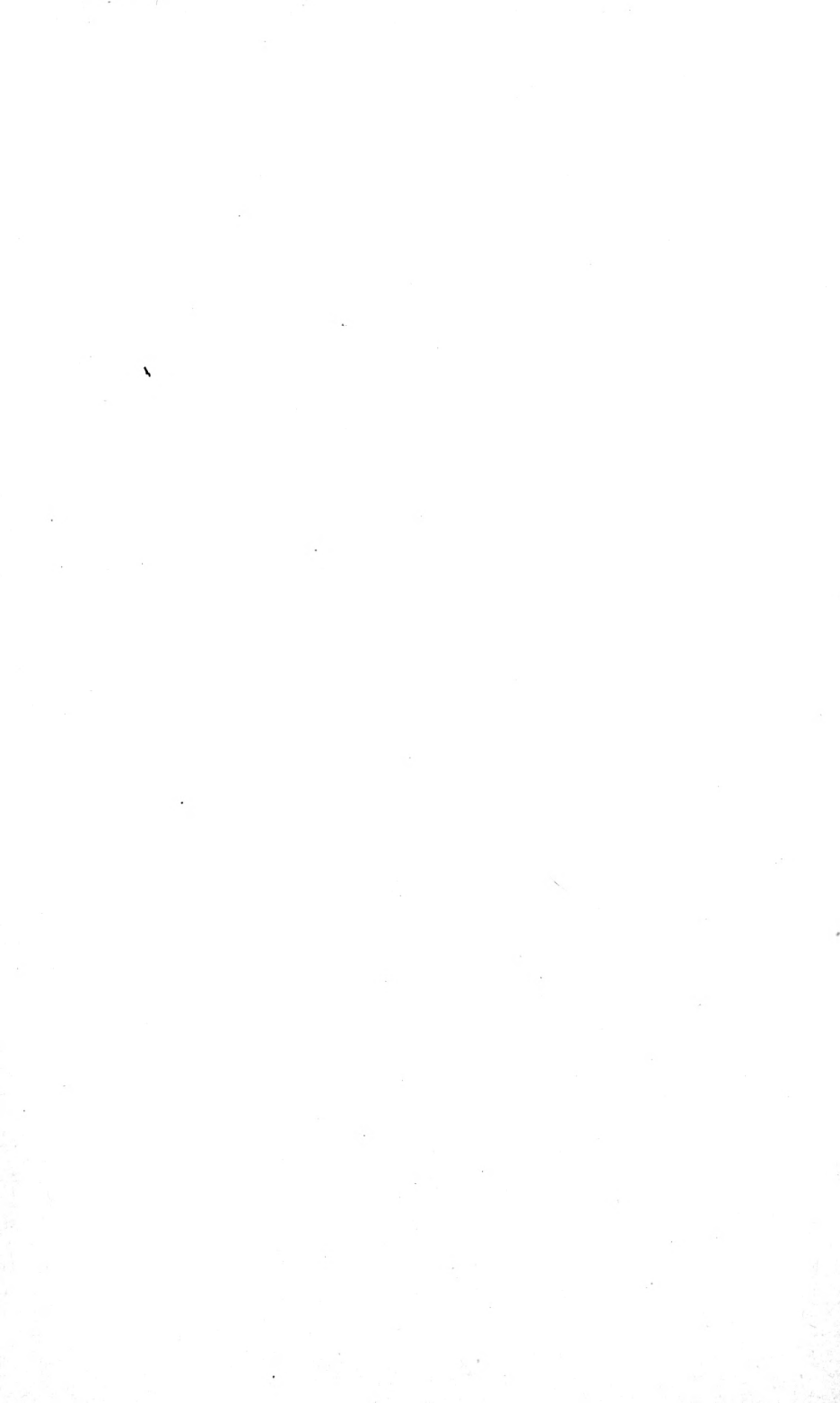
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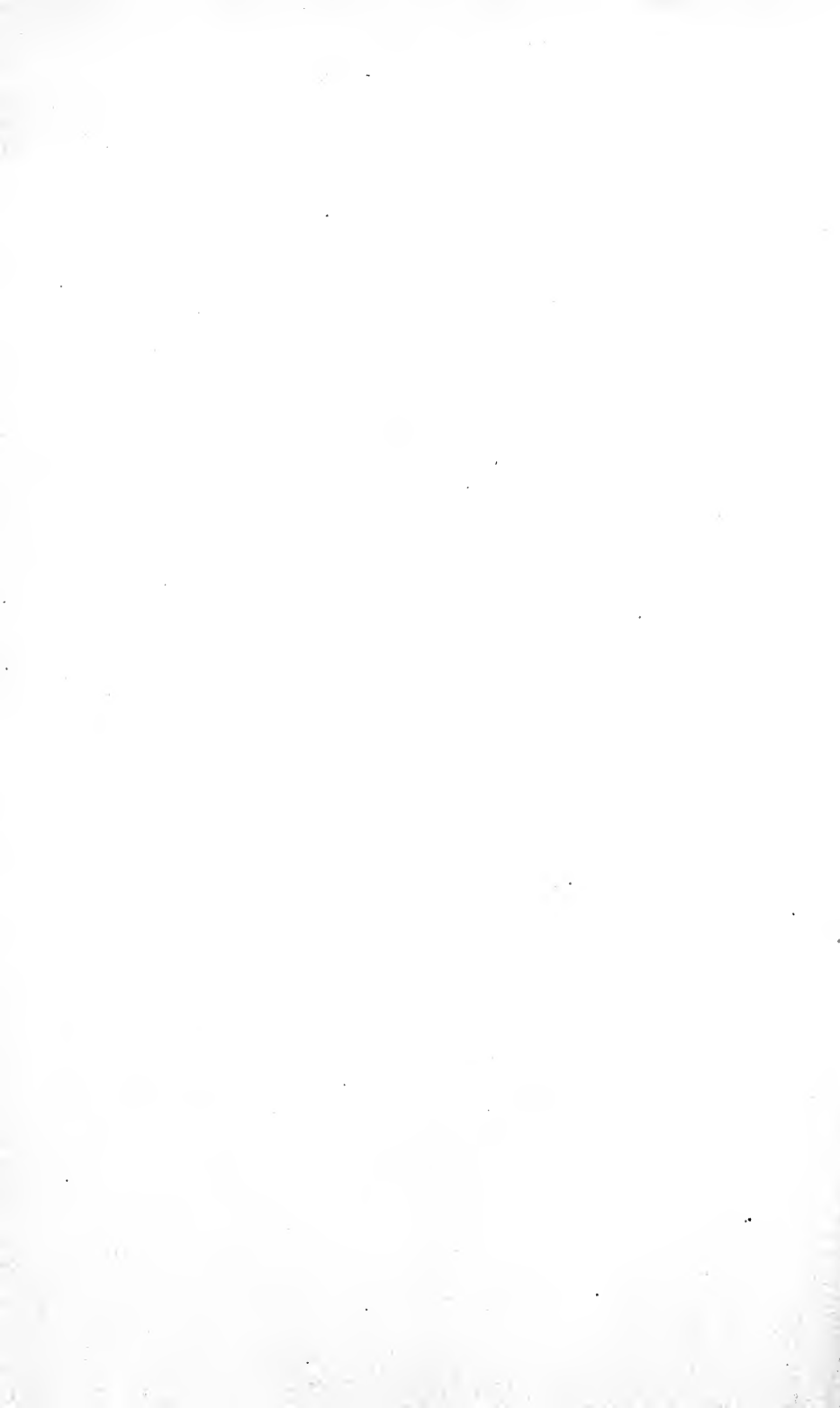
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